Embedded current sheets in the Earth’s magnetotail

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In this investigation we introduce and discuss quantitative parameters of a thin current sheet embedded in the background plasma sheet. We use Cluster statistics and empirical models, as well as self-consistent simulations, to understand the formation and development of embedded current sheets, in particular in the course of substorms. Data and theory show that the embedded sheet thickness is of the order of a proton larmor radius, a constraint equivalent to magnetic flux conservation. The embedded sheet can be essentially described by two dimensionless parameters \( B_0/B_{\text{ext}} \) and \( F_0/F_{\text{ext}} \). \( B_0 \) is the magnetic field at the embedded sheet boundary, \( B_{\text{ext}} \) is the field at the boundary of the background plasma sheet, and \( F_0 \) and \( F_{\text{ext}} \) are magnetic flux values. During the growth phase current density in embedded sheet and \( B_0 \) increase, while thickness decreases. Sheets with the most intense currents (large \( B_0 \)) are observed after onset. The self-consistent anisotropic sheet model, including both electron and proton currents and combined with the Harris-type background shows that when the proton-scale embedded sheet becomes sufficiently thin, an electron-scale current sheet can appear inside it due to enhanced electron curvature drift.


1. Introduction

The magnetotail is the main reservoir of solar wind energy in the Earth’s magnetosphere. In particular during substorms the tail passes through the cycles of loading and unloading of the open magnetic flux [e.g., Baker et al., 1996]. Changes of magnetic configuration are self-consistently reflected in the cross-tail electric current that flows inside the plasma sheet, the region filled with closed field lines and hot plasma. The term “current sheet” is often used when one wants to stress magnetic rather than plasma structure. Instabilities of an intensified current sheet (leading to reconnection or current disruption) are widely believed to be a primary cause of substorm onsets, which initiate explosive conversion of accumulated magnetic energy into particle energy [e.g., Lui et al., 2008; Angelopoulos et al., 2008].

In the basic Harris [1962] current sheet model, vertical (along the sheet normal) scales of plasma density and electric current density are the same. With a single spacecraft current density can be only indirectly estimated. With two properly spaced spacecraft, current density can be computed assuming some orientation of a planar sheet. Already with the early spacecraft observations (ISEE-1,2, etc.) it was noticed that the current sheet often has much smaller vertical scale than the plasma sheet [McComas et al., 1986; Sergeev et al., 1993, and references therein]. While the plasma sheet can be several Earth radii thick, the current sheet scale was often found to be several thousand or even several hundred kilometers, of the order of proton larmor radius. Thus at least in some situations, the thin current sheet is embedded in the plasma sheet background. In particular, thin sheets form and intensify during substorm growth phase and are often observed in association with reconnection activity [e.g., Baumjohann et al., 1992; Asano et al., 2003; Petrukovich et al., 2007; Runov et al., 2008].

The four-spacecraft Cluster mission provided regular quantitative observations of the magnetotail current sheet. It became possible to determine orientation of the sheet, current density vector as well as reconstruct sheet inner structure [e.g., Runov et al., 2006; Nakamura et al., 2006; Baumjohann et al., 2007; Sharma et al., 2008]. It was shown that embedded current sheets are rather ubiquitous in the magnetotail, while the pure Harris-type profiles are rare. Typical scales of current sheets (along the normal) were found to be of the order of several thousand kilometers and current density was up to an order of magnitude higher than that in the surrounding sheet, while density increase was typically \( \sim 10–20\% \) only.

Theory has shown that a thin embedded current sheet with the thickness of the order of ion larmor radius can be self-consistently formed by ions on transient (or “Speiser” [Speiser, 1965]) trajectories [Eastwood, 1972; Francfort, 1972;...]}
Profiles of the current density against magnetic field units. Under this assumption for 24 crossings are also makes electric current density proportional to the magnetic field for the Harris current sheet (blue) and an embedded sheet with the background (green) with their observable parameters.

and Pellat, 1976; Zelenyi et al., 2004]. Experimental profiles of current density and ion distribution functions provided by Cluster and THEMIS were shown to agree with the several theoretical models including the certain degree of embedding [Sitnov et al., 2006; Artemyev et al., 2008, 2009, 2010; Zhou et al., 2009].

In this investigation we introduce general quantitative characteristics of a current sheet embedded in the plasma sheet and discuss their evolution during substorms. The Harris model sheet (Figure 1, blue curve) can be fully described with two parameters: maximal (lobe) magnetic field $B_{ext}$, which is calculated, assuming vertical pressure balance, and maximal current density $j_{\text{max}}$, measured by the Cluster tetrahedron. The embedded sheet model (Figure 1, green curve) needs two extra parameters: the magnetic field at the thin sheet boundary $B_0$, which is also often readily observable and the current density at $B_0$. This final parameter is usually too small to be reliably detected (in the cases when $j_{\text{max}}$ is well resolved) and thus the scale of the background plasma sheet remains unknown.

In the next sections we describe statistics of Cluster observations in terms of these parameters and construct a simple empiric model of an embedded current sheet. We also use a variant of the thin anisotropic current sheet (TACS) model [Zelenyi et al., 2004, 2006] to self-consistently simulate development of embedded sheet.

2. Experimental Data

An extensive selection of Cluster thin single-peaked horizontal current sheet crossings during 2001 and 2004 (with typical downtail distance of observation 16–20 $R_E$) is available from the earlier papers [Runov et al., 2006; Artemyev et al., 2008, 2010]. In this investigation we used a subset of these observations, for which current density maxima and $B_0$ can be determined reliably. Cluster separation ~1000 km during these years is optimal for studies of the so-called ion-scale sheets with thickness of the order of several thousand km. Much thinner electron-scale sheets, registered by Cluster during 2003 [Nakamura et al., 2006] are beyond the scope of this investigation.

We used proton moments from CIS/CODIF instrument [Reme et al., 2001] and magnetic field from FGM instrument [Baugh et al., 2001]. All data were taken from Cluster Active Archive. The lobe magnetic field $B_{ext}$ was determined using the standard assumption of the vertical pressure balance (lobe magnetic pressure is equal to the total pressure inside a sheet) [Baumjohann et al., 1990]. Current density was determined by the curlometer technique [Dunlop et al., 1988; Robert et al., 1998]. The quality of current density measurements (in particular determined using the divB/curlB parameter) is rather high for these data and was extensively discussed elsewhere [Runov et al., 2005; Petrukovich et al., 2007]. $B_0$ was estimated following Artemyev et al. [2010] as a value of the maximum variance magnetic field component at the embedded sheet boundary.

The statistics of $B_0$ and $j_{\text{max}}$ for 24 crossings are presented in Figure 2a. Current density generally increases with $B_0$, but the scatter is large. In Figure 2b the same data are replotted in the normalized coordinates $B_{ext}/B_0$ and $(c/4\pi B_0 j_{\text{max}})/R_0 = z_0/R_0$, where $R_0$ is proton gyroradius in the field $B_0$.

According to Figure 2b the sheet scale appears to be equal to 1–3 proton larmor radii for the range of $B_{ext}/B_0$ observed in our statistics. Such value is in agreement with theory [e.g., Francfort and Pellat, 1976; Zelenyi et al., 2004] and previous estimates [Sergeev et al., 1993]. Further examination of thickness distribution in Figure 2b (e.g., trends) requires much larger amount of data and modeling support. A number of reasons could contribute to the scatter and trends in Figure 2b: (1) difference between temperature of the current carriers and the bulk of the plasma, affecting $R_0$ [e.g., Zhou et al., 2009; Artemyev et al., 2009]; (2) presence of oxygen making the sheet thicker [e.g., Zelenyi et al., 2006]; (3) uncertainties in $B_0$ definition; (4) at large $B_0$ large currents could be underestimated and thickness overestimated because such sheets are narrower than the Cluster tetrahedron [Runov et al., 2005]; (5) both abscise and ordinate in Figure 2b are normalized by $B_0$, and possible systematic errors in this parameter can affect trends.

Since $z_0$ is of the order of proton larmor radius $R_0$ over the range of observed $B_0$, we make our core assumption on the class of embedded sheets of interest in this study that the sheet thickness is fixed in $R_0$ units. Under this assumption the magnetic flux (hereafter per unit width) in a thin embedded sheet $F_0 \sim B_0 z_0$ does not depend on $R_0$ ($R_0$ is inversely proportional to $B_0$) and can be considered as constant. When such embedded sheet “grows,” $B_0$ increases, $j_{\text{max}} \sim B_0^2$, but the thickness becomes smaller $z_0 \sim 1/B_0$. The experimental distribution in Figure 2a does not readily reveal the $j_{\text{max}} \sim B_0^2$ dependence, most likely due to variations in plasma conditions between measurements. The relation $j \sim B^2$ also makes electric current density proportional to the number density of current carriers (similar to the Harris model, assuming constant temperature and drift velocity). Using the scatter in Figure 2b $z_0 \sim (1–3)R_0$ and depending on the details of the sheet profile (see section 3.1) one can get an estimate $F_0 = (1 - 5)B_0 R_0$. 

Figure 1. Profiles of the current density against magnetic field for the Harris current sheet (blue) and an embedded sheet with the background (green) with their observable parameters.
The same is true for the whole plasma sheet, if the total closed magnetic flux $F_{\text{ext}}$ (per unit width) at a given downtail distance is conserved. The nature of flux conservation is different for the plasma sheet and the embedded current sheet. The plasma sheet contains the closed magnetic field lines, distinctly different from the open field lines. However, there are no predefined magnetic field lines, belonging to the thin embedded sheet. Conservation of flux (per unit width) assumed in this study is related to the specific sheet formation mechanism, setting its scale at the proton larmor radius.

The value of $B_0 R_0$ is 0.006 Wb/m (for temperature 4000 eV). $F_0$ then is 0.006–0.03 Wb/m, at least factor of 10 smaller than the estimate of the total closed magnetic flux $F_{\text{ext}} \sim 0.4–0.5$ Wb/m at the downtail distances around 15 $R_E$ [e.g., Petrukovich et al., 1998]. As we will show below this large difference imposes an important constraint on the embedded current sheet.

3. Empiric Model of Embedded Sheet

The hyperbolic Harris-type profile has infinite spatial range, so that plasma density vanishes at infinity. This feature is typical for available self-consistent models. However it is not always convenient, for example such a sheet has infinite magnetic flux and energy of magnetic field. Therefore in this section we introduce several non-self-consistent but more practical ad hoc profiles, which are spatially finite and use them to construct the embedded sheet. This exercise also helps to understand basic aspects of embedded sheet formation.

3.1. Definition of Sheet Profiles

We consider a sheet with maximal magnetic field $B_m$, maximal current density $j_m$, and spatial scale $z_m = (c/4\pi)B_m/j_m$. The full sheet thickness $z_m^* (j(\pm z_m^*) = 0)$ varies depending on profile. The magnetic flux per unit width (within $0 < z < z_m^*$) is $F_m = \int B dz$. It is convenient to define normalized profiles using dimensionless variables $\tilde{z} = z/z_m^*$, $\tilde{B} = B/B_m$, $\tilde{j} = j(B_m/z_m^*)(c/4\pi)$. Then the dimensionless constants $z_m^*/z_m$ and $\tilde{F}_m = (F_m/B_m)/z_m^*$ characterize particular form of a normalized profile.

In this general case $\alpha \neq 0$, the uniform limit $j = \text{const}$ is achieved with $\alpha \to \infty$. Larger $\alpha$-s create thinner sheets (in terms of $z_m^*$) (Figure 3a). The variants with $\alpha < 1 (z_m^* > 2)$ have somewhat unphysical cusp-type maximum at $\tilde{z} = 0$ (not shown here). A parabolic case of $\alpha = 2$ results in $z_m^* = 1.5$ and $\tilde{F}_m = 15/16$. For a linear case $\alpha = 1$, $z_m^* = 2$ and $\tilde{F}_m = 4/3$.

Another choice is a modified Harris solution, in which the current is forced to vanish at a finite distance from the center [Veltri et al., 1998]. However to fix $\tilde{B}_m = 1$ and $\tilde{j}_m = 1$, one needs to introduce an additional parameter $\lambda^*$, adjusting the spatial scale:

\[
\tilde{B} = \lambda^* \frac{\tanh(\tilde{z}/\lambda^*) - (\tilde{z}/\lambda^*) \cos^{-2}(\tilde{z}_m^*/\lambda^*)}{\tanh(\tilde{z}_m^*/\lambda^*) - (\tilde{z}_m^*/\lambda^*) \cos^{-2}(\tilde{z}_m^*/\lambda^*)} \cos^{-2}(\tilde{z}/\lambda^*) - \cos^{-2}(\tilde{z}_m^*/\lambda^*)} \tanh(\tilde{z}_m^*/\lambda^*) - (\tilde{z}_m^*/\lambda^*) \cos^{-2}(\tilde{z}_m^*/\lambda^*)}
\]

\[
\tilde{j}(\tilde{z}) = 1/\lambda^* \frac{\cos^{-2}(\tilde{z}/\lambda^*) - \cos^{-2}(\tilde{z}_m^*/\lambda^*)}{\tanh(\tilde{z}_m^*/\lambda^*) - (\tilde{z}_m^*/\lambda^*) \cos^{-2}(\tilde{z}_m^*/\lambda^*)} \cos^{-2}(\tilde{z}/\lambda^*) - \cos^{-2}(\tilde{z}_m^*/\lambda^*)} \tanh(\tilde{z}_m^*/\lambda^*) - (\tilde{z}_m^*/\lambda^*) \cos^{-2}(\tilde{z}_m^*/\lambda^*)}
\]
at large $\tilde{z}$ and are undistinguishable in the real data. For $\tilde{z}_m^* = 1.5$ the profile becomes exactly parabolic and $\lambda^* = \infty$. Values of $F_m$ and $\lambda^*$ for some $\tilde{z}_m^*$ are in Table 1.

[21] The requirements $B_0 = 1$ and $j_0 = 1$ rather strongly limit possible shape, and the degree of sheet slenderness $\tilde{z}_m^*$ is practically the only remaining free parameter. Since the most of the magnetic flux is contained at large $\tilde{z}$, the total flux content depends strongly on $\tilde{z}_m^*$. The difference in flux between $\tilde{z}_m^* = 1$ and $\tilde{z}_m^* = 2.5$ is of the factor of four. The value $F_m$ defines the correction factor, used in section 2 to estimate magnetic flux in the embedded sheet $F_0$.

[22] Comparison with the sample sheet taken from the TACS model suggests that our profiles with $\tilde{z}_m^* \approx 1.5$–2 are the best (Figure 4a). With the power law profiles many calculations can be done analytically. The Harris-type profiles require numerical integration but are closer to a self-consistent solution. A rather broad variety of embedded sheet profiles was observed by Cluster mission. Figure 4b presents two examples which shapes are close to a power law with $\alpha = 1$, $\tilde{z}_m^* = 2$ (more triangular) and $\alpha = 4$, $\tilde{z}_m^* = 5/4$–$7/6$ (more table-like) (figure adopted from Artemyev et al. [2008]).

### 3.2. Model of Embedding

[23] With the profiles described in section 3.1 and the flux conservation assumption for both the thin inner sheet and the whole plasma sheet we build a model of embedding. The system will be described in terms of magnetic field and magnetic flux, which proved to be the most suitable parameters. The magnetic field is a convenient observable parameter, while the magnetic flux content of both thin and background sheets can be considered as (almost) constant (e.g., during substorm growth phase) contrary to thickness and current density, which can change by an order of magnitude.

[24] The input parameters of the embedded system are the ratios $F_0/F_{ext}$ and $B_0/B_{ext}$, defining the relative sheet sizes and strengths. Here the subscript “0” refers to the values at the boundaries of embedded sheet and the subscript “ext” to that at the boundary of the background (plasma) sheet.

[25] Our algorithm has three major steps.

[26] 1. We choose the desired dimensionless profiles (according to section 3.1) for both embedded and background sheets. Details of the profiles are described by the respective $\tilde{z}_m^*$ and $F$ parameters.

[27] 2. The profiles are rescaled (as explained below) according to input requirements.

[28] 3. The final magnetic and current density profiles are obtained as the sums of embedded and background curves. One particular example is illustrated in Figure 5. Here the modified Harris sheet with $\tilde{z}_m^* = 2$ was used to generate both the inner and outer sheets. The embedded, background and final magnetic field profiles are in Figure 5a. The final current density profile is in Figure 5b (green curve).

[29] The maximal magnetic fields created by currents of embedded and background sheets are $B_1$ and $B_{ext} - B_1$, respectively. Note that $B_1 \neq B_0$, $B_0$ (see equation (3)) is the sum of $B_1$ and the magnetic field of the background sheet at the boundary of the embedded sheet $\tilde{z}_m^*$ equal to $(B_{ext} - B_1)$ $f_{ext} (\tilde{z}_m^*)$, where $f_{ext}$ is the function defining the background sheet profile.

[30] It is convenient to normalize the spatial scales of the embedded sheet $\tilde{z}_0$ and of the modified background sheet $\tilde{z}_{ext}$ with respect to the scale of the background sheet with no embedded sheet $\tilde{z}_{ext}^{ini}$, assuming that the total magnetic flux $F_{ext}$ is constant. Such a normalization allows us to study the evolution of sheets, comparing cases with different $B_0$. The thickness of the embedded sheet is defined in equation (4). The flux conservation assumption for the whole system

<table>
<thead>
<tr>
<th>$\tilde{z}_m^*$</th>
<th>$\tilde{F}_m$</th>
<th>$\lambda^*$</th>
</tr>
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<td>2.5</td>
<td>1.85</td>
<td>1.077</td>
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<td>1.8</td>
<td>1.20</td>
<td>1.457</td>
</tr>
<tr>
<td>1.7</td>
<td>1.11</td>
<td>1.69</td>
</tr>
<tr>
<td>1.6</td>
<td>1.03</td>
<td>2.255</td>
</tr>
</tbody>
</table>

Figure 3. Profiles of the current density ($\tilde{j}$) against spatial scale ($\tilde{z}$) of the (a) power law empirical models and (b) modified Harris sheet empirical models.
gives \( F_{\text{ext}} = \tilde{F}_{\text{ext}} B_{\text{ext}} z_{\text{ext}} = \tilde{F}_{\text{ext}} (B_{\text{ext}} - B_1) z_{\text{ext}} + \tilde{F}_0 B_1 z_0 + B_1 (z_{\text{ext}} - z_0^\text{ini}) \). After some simple algebra we get equation (5) for \( z_{\text{ext}} \).

Thus we have three coupled equations, defining all parameters of an embedded system:

\[
B_0 / B_{\text{ext}} = B_1 / B_{\text{ext}} + (1 - B_1 / B_{\text{ext}}) f_{\text{ext}}(z_0^\text{ini}) \quad (3)
\]

\[
z_0 / z_{\text{ext}}^\text{ini} = (F_0 / B_0) / ( (F_{\text{ext}} / B_{\text{ext}}) / \tilde{F}_{\text{ext}} )
= (F_0 / F_{\text{ext}}) (B_{\text{ext}} / B_0) (\tilde{F}_{\text{ext}} / \tilde{F}_0) \quad (4)
\]

\[
z_{\text{ext}} / z_{\text{ext}}^\text{ini} = 1 + (F_0 / F_{\text{ext}}) (B_1 / B_0) \left( \frac{z_{\text{ext}}}{\tilde{F}_{\text{ext}}} - 1 \right) / \left( 1 + B_1 / B_{\text{ext}} (z_{\text{ext}} / \tilde{F}_{\text{ext}} - 1) \right) \quad (5)
\]

The ratio of the current density created by the embedded sheet \( j_1 \) and initial background current density \( f_{\text{ext}} \) is

\[
\frac{j_1}{f_{\text{ext}}} = \frac{B_1 / z_0}{B_{\text{ext}} / z_{\text{ext}}^\text{ini}} = \frac{\tilde{F}_0}{F_{\text{ext}}} \frac{F_{\text{ext}}}{F_0} \frac{B_0 B_1}{B_{\text{ext}}}. \quad (6)
\]

The maximal possible current density (when \( B_1 = B_0 = B_{\text{ext}} \)) is

\[
(\tilde{F}_0 / \tilde{F}_{\text{ext}}) (F_{\text{ext}} / F_0).
\]

In comparison we compute also a self-consistent Harris-type solution with two plasma components, having the different flow velocities and the same temperatures, following, e.g., Liu et al. [2010] (Figure 5b, blue curve). It is rather close to our empirical model (green curve) and thus our model is not so far from a self-consistent solution.

Figure 4. (a) Current density \( j \) against spatial scale \( z \) for the two empirical models and the TACS model and (b) current density against \( Z \) GSM (centered on the current density maximum) for two current sheet crossings by the Cluster spacecraft (adapted from Artemyev et al. [2008]).

Figure 5. (a) Magnetic field against spatial scale for embedded and background components of non-self-consistent sheet model and (b) the current density against spatial scale for the non-self-consistent sheet model (green) and the self-consistent two component Harris sheet model (blue). Notations are explained in text.
uncertainty of our knowledge of $F_{\text{ext}}$ and $F_0$ it is still reasonable to use the estimate of equation (7). At the Cluster downtail distances typically $F_0/F_{\text{ext}} \leq 0.1$ (section 2) and the minimal $B_0/B_{\text{ext}} \approx 0.2$–0.3. This estimate is in a good agreement with the statistics in Figure 2b.

[36] Lets consider now the opposite case, $B_0 \approx B_{\text{ext}}$. If $F_0 \ll F_{\text{ext}}$ an embedded sheet is very thin relative to the background one. Outside it magnetic field is almost constant $B_0 \approx B_{\text{ext}}$ and plasma pressure is negligible. Plasma is contained only in the narrow stripe of embedded sheet. Since in a quasi-stationary state (growth phase) formation of an embedded sheet is generally starting in the thick background plasma sheet keeping a lot of plasma, such a configuration can be considered as rather extreme one. Thus during the most of growth phase $B_0/B_{\text{ext}}$ should be likely small and closer to $\sqrt{F_0/F_{\text{ext}}}$.

[37] At the Cluster downtail distances $F_{\text{ext}}$ becomes smaller (making large $B_0/B_{\text{ext}}$ more probable) after reconnection event and plasmoid ejection, when the most of plasma and/or closed magnetic flux are evacuated from the plasma sheet. In our statistics there are 10 cases with $B_0 > 0.5B_{\text{ext}}$ (Figure 2b) and eight of them were indeed registered after substorm onsets. Figure 6 contains an example of such an event, registered on 8 October 2001. It is a prolonged interval of intermittent tailward and Earthward flows (reconnection events) after the onset of a strong substorm ($AL \approx -500$ nT). Cluster satellites stay in the vicinity of a thin current sheet, revealing itself by the differences between $B_z$ curves. Three fast crossings, allowing to determine the thin sheet profile occurred at 1249, 1307, and 1308 UT. The lobe magnetic field ($B_{\text{ext}}$) at the moment of first plasmoid ejection at 1250 UT was equal to 37 nT. Later it decreased to 30 nT, in agreement with the anticipated pressure depletion in the course of expansion phase. $B_0$ at the studied crossings were estimated as 22–25 nT. Such $B_0$ can be also readily discerned visually in Figure 6 as $B_z$ value at which the difference between the observations from the different spacecraft becomes small. Thus $B_0/B_{\text{ext}}$ is equal to $\sim 0.6$ in the beginning of the interval and increases to $\sim 0.8$ at the end of the interval, after the reconnection event.

[38] The configuration with $B_0 \approx B_{\text{ext}}$ should be more typical also in the distant tail where the total plasma sheet flux $F_{\text{ext}}$ is small.

4. Self-Consistent Model of Embedded Current Sheet

[39] In this section we apply a self-consistent model of a thin embedded sheet to verify conclusions of section 3.3 and investigate formation of sheets with varying intensity. Our model is based on a combination of 1-D model of anisotropic current sheet by Zelenyi et al. [2004, 2006] and an isotropic current sheet with finite normal component of magnetic field. The first part is responsible for the thin sheet and the second one is responsible for the background sheet. Our approach is close to that of Zhou et al. [2009], but in our model the normal component of the magnetic field $B_z$ is directly taken into account. It is necessary to note that formally the one-dimensional Harris-type isotropic equilibrium is not valid with finite $B_z$ and a two dimensional model is necessary [e.g., Lembege and Pellat, 1982]. However the dependence on $X$ is rather weak and if one is interested in a
one-dimensional vertical cut only, it is reasonable to simplify the model, using the standard 1-D Harris solution.

[40] We compute a set of self-consistent profiles of current sheet with different layers of embedding as snapshots of different stages of its evolution. We suppose that change of current sheet state during its evolution is much slower than particle large-scale rotation in the normal magnetic field $B_z$.

[41] The magnetic field has two components $B_x(z)$ and $B_z$. In the TACS model plasma consists of magnetized electrons and nonadiabatic ions, which move along field lines in and out the sheet and have anisotropic pressure distribution, so that the action integral $I_z = (m/2\pi)\int v^2 dz$ during their motion is approximately conserved. These transient Speiser particles carry the current in a thin embedded sheet. The electron motion along the field lines is assumed to be fast enough to support a quasi-equilibrium Boltzmann distribution in the presence of an electrostatic potential. Electron equations of motion are considered in a semifluid approach. The sheet thickness appears in this model self-consistently. The full description of the model is available elsewhere [Zelenyi et al., 2004, 2006]. The ion distribution at the boundary is defined as:

$$ f_i(z, v) = \frac{n_0}{(\sqrt{\pi} v_{TI})^3(1 + \text{erf}(\varepsilon^{-1}))} \sum_{s=1,2} \exp \left\{ -\frac{(v_i + (-1)^s v_{TI})^2 + v_{zf}^2}{v_{TI}^2} \right\}, $$  

(8)

[42] Here $n_0$ is the total density, $v_{TI}$ is the thermal ion velocity, $v_{zf}$ is the drift velocity at the boundary, and indices $s = 1, 2$ correspond to plasma flows from two edges of current sheet. $\varepsilon = v_{TI}/v_{D1}$ is a free parameter of the model, characterizing the anisotropy of ion flows outside current sheet. The distribution function inside the sheet is obtained self-consistently, basing on conservation of $I_z$ and magnetic profile.

[43] The background plasma component consists of hot particles with isotropic distribution, creating only diamagnetic current [Harris, 1962]. The thickness is regulated by the boundary condition and the drift velocity.

$$ f_s = n_0 (\sqrt{\pi} v_{T2})^{-3} \exp \left[ \frac{(v_i/v_{D2})^2}{v^2} \right] \cdot \exp \left\{ -\frac{v_i^2 + (v_i - v_{D2})^2 + v_z^2}{v_{T2}^2} \right\} $$

(9)

Here $T_2$, $v_{T2}$, $v_{D2}$ are temperature, thermal velocity and drift velocity of the isotropic component, $A_x$ is a component of vector potential.

[44] Finally, the Ampere’s law is

$$ dB_i/dz = (4\pi/c) \int v_i f_i(z, v) d^2v + (1 - n_i) \int v_i f_s(z, v) d^2v + j_{ex}(z), $$

$$ B_i(z)_{z=\pm \infty} = B_{ext} $$  

(10)

Here $n_i$ is the coefficient characterizing the relative density of anisotropic protons in comparison with isotropic ones; this value is varied from 0 to 1. Here $j_{ex}$ is the electron current. Numeric plasma parameters were taken as thickness of the outer sheet $L/R_0 = 20$ ($R_0$ is the ion Larmor radius), $\varepsilon = v_{TI}/v_{D1} = 0.3$, $T_1/T_2 = 5$, $T_1 = T_2 = 4$ keV, $B_{ext} = 20$ nT, $B_z = 2$ nT.

[45] Figure 7 shows a set of current density profiles $j_1$ as a function of $B_i/B_{ext}$ for different $n_i$. Figure 8 shows the same profiles with respect to a spatial coordinate $z$ for the central region $|z| < 8000$ km in which details of a narrow maximum can be discerned (Figure 8a) and for $|z| < 24000$ km (Figure 8b). The stages of sheet evolution are best seen in Figure 7.

[46] The case $n_i = 0.0$ corresponds to completely isotropic current sheet with the scale ~40000 km. At $n_i$ from 0.05 to 0.5 a new sheet with a smaller scale ~2000 km appears on the top of background due to formation of the proton-dominated anisotropic current sheet. The minimal magnetic field at the boundary of appearing embedded sheet ($B_0$) is 0.25–0.3 of $B_{ext}$. While the current density grows, $B_0$ increases and the proton embedded sheet becomes thinner. At larger $n_i$, $L/R_0$ ~ 0.2–0.3 a third scale ~200 km appears inside it, corresponding to the electron curvature currents [Zelenyi et al., 2004]. At $n_i ~ 0.5$ when the proton sheet is thin enough, electron current becomes larger than the ion one. Thus one more level of embedding forms with very thin and intense electron-dominated sheet. These electron currents are proportional to inverse curvature radius of magnetic field and electron anisotropy.

[47] Such model behavior is in agreement with our empiric picture. The more detailed analysis of this simulation will be published elsewhere. Though the self-consistent model is generally more powerful, the empirical model (section 3) is more flexible and allows direct control over all sheet parameters. For example magnetic flux cannot be calculated.
exactly in the self-consistent model because the sheet is spatially infinite.

5. Discussion

5.1. Validity of the Model

[48] We suggest that the magnetotail plasma sheet can be described with a simple scheme including the thin current sheet embedded in the much thicker background plasma sheet. The embedded current sheet in many cases has characteristic thickness similar to the local proton larmor radius, a condition equivalent to conservation of the magnetic flux (per unit length) in it. The same assumption of constant magnetic flux can be applied also to the whole plasma sheet. Thus the number of free parameters describing the embedded system can be reduced. Then the pair of sheets can be conveniently parameterized using the ratios \( B_0/B_{\text{ext}} \) and \( F_0/F_{\text{ext}} \). The introduction of the finite ad hoc sheet profiles allows to calculate exactly many parameters as well as to understand the sheet evolution during growth phase, when embedded current intensifies.

[49] To track evolution, our models keep \( B_{\text{ext}}, F_{\text{ext}} \) and \( F_0 \) constant. In the real plasma sheet these parameters do change. Moderate violations of flux conservation do not affect validity of our main conclusions, since for the most of Cluster data \( F_0/F_{\text{ext}} \ll 1 \). Nevertheless we discuss briefly possible causes of such changes below.

[50] \( B_{\text{ext}} \) is often increasing in the course of growth phase. However, especially during smaller substorms, intensified current sheet can develop with only minimal change of \( B_{\text{ext}} \) [e.g., Petrukovich et al., 2000].

[51] The change of the closed magnetic flux \( F_{\text{ext}} \) (at a given \( X \) GSM) can be driven by at least four processes: (1) The magnetic stretching moving closed magnetic field lines outward and increasing \( F_{\text{ext}} \). (2) The removal of flux round to the dayside by global convection from the tail boundary of dipolar and stretched lines, decreasing \( F_{\text{ext}} \). (3) The addition of closed flux from open flux in the tail via a distant neutral line. (4) The removal of closed magnetic field lines without substorm initiation due to pseudo-breakup level reconnection in the near-Earth tail). \( F_0 \) can change if the thin sheet scale in the units of the larmor radius and/or the form of sheet profile evolve with changing \( B_0 \).

[52] Quantitative estimates of these factors are not available yet. Our model can be easily modified to allow for slow change of \( B_{\text{ext}}, F_0, \) and \( F_{\text{ext}} \) if necessary and if some functional dependence of their evolution is known.

[53] Another major simplification is usage of two current sheets with the separate particle populations and uniform temperatures. The real configuration could be of course more complicated. In addition to these two particle groups a “third” relatively cold plasma component with substantial density is often observed [Liang et al., 2009; Artemyev et al., 2009]. Zhou et al. [2009] and Liu et al. [2010] modeled current sheets from the THEMIS observations, creating background sheets with much colder plasma. Because the cold component has small pressure, \( B_0/B_{\text{ext}} \approx 1 \) and the \( j-B \) profile of such a sheet is very close to a Harris one. Thus such a model alone is not applicable to explain Cluster embedded sheets, which principally require presence of the hot background with dominating pressure (\( B_0/B_{\text{ext}} < 1 \)).

Figure 8. Plots of the current density versus position along the sheet normal from the sheet center for a series of TACS models, showing (a) the inner part of the current sheets and (b) the full width of the current sheets.
In the models particles responsible for the currents in the two sheets belong to two distinctly different populations and the current density is regulated by a change of the number density. Actual mechanisms of current formation can be more complicated. For example protons can migrate between populations due to local acceleration and/or scattering processes (see discussion in the work of Artemyev et al. [2010]).

Finally, the approach with the single free parameter $B_0/B_{ext}$ and constant magnetic flux assumption proved to be sufficient to perform comparison with models and to explain many observational facts [see also Artemyev et al., 2008, 2009, 2010; Zelenyi et al., 2010].

5.2. Maximal Current Density

Sheets with larger current densities are of the special interest, since they are believed to be more susceptible to instability. In our model $j_0$ depends on $B_0$, while the latter is controlled via $B_0/B_{ext}$ and $F_0/F_{ext}$.

In the distant tail $B_0/B_{ext}$ is expected to be close to unity, since the total plasma sheet magnetic flux is small. However, $B_{ext}$ here is also smaller (of the order of 5–10 nT) and thus current density should be moderate $\sim 1$–2 nA/m². A single spacecraft estimate of Pulkkinen et al. [1993] gives a similar value.

In the nearer tail current density can be 10–20 nA/m² (Figure 2a), but more intense sheets should be more common after onsets, when large $B_0/B_{ext}$ is more probable. As such, our conclusion is consistent with the statistics of section 2 as well as other observational results, stating that sheets are thinner after substorm onsets [Baumjohann et al., 1992; Asano et al., 2003; Petrukovich et al., 2007].

Of particular interest is the question of where in the near tail current density maximizes during growth phase. In such a case one should expect moderate $B_0/B_{ext} < 0.5$. The total cross-tail current is increasing toward Earth, but the change of current density depends on many parameters. $F_{ext}$ and $B_{ext}$ are both increasing Earthward and the result of their interplay is not obvious in the frame of our approach. Comparison of Cluster observations taken before onsets closer and farther than 16 $R_E$ did not reveal any substantial current density difference [Petrukovich et al., 2009]. An investigation with the later THEMIS and Cluster data (taken closer to Earth) is necessary to solve this problem.

Sheet parameters also depend on outer conditions, such as solar wind dynamic pressure. In particular Sergeev et al. [1993] described the ISEE observation with much stronger current density than in our Cluster statistics. ISEE spacecraft were $\sim 5$ $R_E$ closer to Earth than the typical Cluster distance. The solar wind pressure was 4–5 nPa in the ISEE case, significantly larger than in our examples and implying larger current.

5.3. Plasma Sheet Volume (Thickness)

Assumption of constant flux in the plasma sheet suggests that the plasma sheet thickness decreases when $B_0$ increases (equation (5)). When $F_0 \ll F_{ext}$ and $B_1 \approx B_0 \approx B_{ext}$ the sheet shrinks by a factor of $z_{ext}/F_{ext} \sim 2$. This plasma sheet thinning proceeds only due to the embedded sheet intensification rather than due to external pressure increase.

For a typical growth phase $B_0/B_{ext} \approx 0.3–0.5$ the plasma sheet thickness decreases by a factor of 0.8–0.9. Thus the lobe volume and hence the open magnetic flux can increase even with the constant $B_{ext}$ and solar wind pressure. This aspect could be important for the models, in which the open flux estimates are made basing on local pressure measurements [Shukhtina et al., 2009, and references therein].

5.4. Criterion of Being Inside the Current Sheet

Immediately after the onset of an activation (e.g., reconnection) in the plasma sheet, plasma flows can be weak and localized to vicinity of the neutral sheet, since initially reconnection involves only inner magnetic field lines. Registration of such weak flows is important to determine the onset location (whether the initial flow is tailward or Earthward). When a spacecraft is located inside the thin embedded sheet, its position is reliably within an ion gyroradius with respect to the neutral plane and thus the initial plasma flow will not be missed. When the spacecraft is outside the embedded sheet, its closeness to the neutral plane is not guaranteed and cannot be judged by the local magnetic field value only. In particular, an example of Petrukovich et al. [2009], Figure 1 shows that the tailward flow was already unobservable at $B_2 \approx 15 – 20$ nT ($\beta \approx 2–4$). The boundary of an embedded current sheet during growth phase in the frame of our model is at $\beta = (B_{ext}/B_0)^2 – 1 \sim 4 – 10$. However often the $\beta \approx 1$ rule is implemented to qualify spacecraft location inside the plasma sheet in a position to observe plasma flows.

5.5. Evolution of Embedding During Growth Phase and Onset

In the model formation and development of embedded sheet can be simulated if the density of the respective current carriers (protons with high drift velocity) is increased at the expense of the density of the background sheet. Thus the evolution of the sheet is studied as a sequence of states.

In experiment, sheet thinning during growth phase can be traced as change of $B_0$. To detect it, a series of fast crossings is necessary, which is rather improbable. Therefore a number of available examples is small. Sergeev et al. [1993] estimated that $B_0/B_{ext}$ was growing from 0.3 to $\sim 0.5$. Another example also suggests the increase approximately from 0.3 to 0.5 [Petrukovich et al., 2007, Figure 2]. These numbers are consistent with our statistics, described in section 2.

According to the model initially the embedded sheet appears with some minimal $B_0/B_{ext} \sim \sqrt{F_0/F_{ext}} \sim 0.3$ (at the Cluster downtail distances) and the largest thickness. As the number of current carriers grows, the peak current density and $B_0/B_{ext}$ increase, while the sheet becomes thinner. The background sheet shrinks appropriately. The embedded sheet does not change the plasma sheet configuration substantially while $B_0/B_{ext}$ is close to $\sqrt{F_0/F_{ext}}$. The scheme of such sheet evolution was verified with the self-consistent TACS model.

Of particular interest is the ability of our self-consistent model to reproduce appearance of a very intense electron current sheet, when the ion sheet becomes sufficiently thin (and $B_0$ large). Electron current in it is driven by the electron anisotropy and magnetic curvature and thus depends on $B_y$, $B_z$, and electron pressure tensor. In particular, the nonzero $B_y > B_z$ increases curvature radius by a factor of $(B_y/B_z)^2$ and thus is capable to damp the development of electron current. The thinnest sheets with $B_0/B_{ext} > 0.5$ also should form more readily when $F_{ext}$ decreases. With such complications formation of electron current sheets appears to be more probable after onset in reconnection zones, when
plasma is more anisotropic, curvature is stronger, $B_z$ is smaller, $B_0$ is larger and the sheet is thinner.

[66] Super-thin electron sheets are almost unobservable by the Cluster tetrahedron with separation of the order of 1000 km (as in 2001 and 2004) because the peak of current density is smoothed. However, since formation of electron sheet is expected only in rather special conditions, such a situation should not happen often.

[69] Stability of a sheet with the Harris profile and finite $B_z$ to the tearing-type perturbation is well established in theory. However stability of an embedded configuration is much less investigated. Burkhart et al. [1992] estimated that embedded sheet with Harris profile and nonadiabatic ions has a maximum of tearing growth rate at $B_0/B_{ext} \sim 1/\sqrt{3}$ in the absence of any electron effects. Zelenyi et al. [2008] suggested in the frame of TACS model that formation of embedded sheet significantly increases the “free energy,” which can overcome the stabilizing effect even in the case of finite $B_z$.

Conclusions

[70] A thin current sheet embedded in the background plasma sheet is frequently observed in the Earth’s magnetotail, in particular during substorms. In the paper we introduce and analyze quantitative characteristics of embedding using Cluster mission statistics, empirical model, and self-consistent simulations. All three approaches give consistent results. The embedding is essentially described by two dimensionless parameters $B_0/B_{ext}$ and $F_0/F_{ext}$. During growth phase current density and $B_0$ increase, while the sheet thickness decreases. The process can be modeled by increasing density of particles responsible for the embedded sheet while decreasing that of the background one and assuming magnetic flux conservation. There exists the minimal possible $B_0/B_{ext} \sim \sqrt{F_0/F_{ext}} \sim 0.3$ (at the Cluster downtail distances). In the course of growth phase $B_0/B_{ext}$ increases, with $B_0/B_{ext} > 0.5$ more typical for the post onset configuration, when $F_{ext}$ decreases. When the ion sheet becomes sufficiently intense, even thinner electron current sheet could appear inside it due to enhanced electron curvature drift. The suggested approach defining embedded sheets can be applied also to other aspects of magnetotail dynamics.

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