Two types of tangential magnetopause current sheets: Cluster observations and theory

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Received 19 May 2011; revised 20 September 2011; accepted 24 September 2011; published 7 December 2011.

Early magnetopause observations revealed that the magnetic field can rotate across tangential current sheets in the form of C-and S-shaped hodograms. We use the four-spacecraft magnetopause crossings by Cluster in order to study the structure of the C- and S-sheets. We show that both current sheets can be described by analytical equilibria. We employ a force-free current sheet equilibrium for description of the C-sheet and develop a new equilibrium to describe the S-sheet. We suggest that both equilibria be used for setting up initial conditions in the next generation of current sheet simulations.


1. Introduction

The outer boundary of the Earth’s magnetosphere, the magnetopause, determines the rate of penetration of the shocked solar wind plasma into the magnetosphere. This rate depends on the embedding conditions and on the magnetopause structure: its current sheet can be stable or can be subject to instabilities.

The magnetopause current sheet can be either a tangential or a rotational discontinuity. Equilibrium conditions and stability of the tangential discontinuities have been intensively investigated by kinetic approach [Galeev et al., 1986; Roth et al., 1996; De Keyser and Roth, 1998].

Theoretical attempts to understand the tangential current sheet physics can be divided into two groups: the equilibria with a single plasma population which is trapped inside the current sheet [Harris, 1962; Alpers, 1969, 1971; Channell, 1976; Mottez, 2003], and the equilibria when plasma is described by different particle populations on the two sides of the current sheet [Lemaire and Burlaga, 1976; Lee and Kan, 1979; Roth et al., 1996]. [Whipple et al., 1984] suggested to generalize these models by utilizing the first adiabatic invariant \( J = \oint \mathbf{v}_z dz \), where \( \mathbf{v}_z \) and \( z \) are velocity and coordinate across the current sheet. Such a generalization has later been done for the magnetotail current sheet [Zelenyi et al., 2006; Sitnov et al., 2006; Zelenyi et al., 2011], and also for the magnetopause current sheet [Artemyev, 2011]. In the above models the force balance is maintained either by the pressure gradient or by the inertia of ions’ motion.

Despite several analytical equilibria are available, only the simplest of them, like [Harris, 1962], were used for setting up initial conditions in kinetic simulations [see, e.g., Pritchett et al., 1996; Daughton, 1999; Silin et al., 2005; Lapenta et al., 2010; Karimabadi et al., 2011, and references therein].

Early in situ magnetopause observations revealed that the magnetic field can rotate across the current sheet in form of C and S letters (hereafter C- and S-sheets) [Berchem and Russell, 1982a, 1982b; Russell, 2003]. The stability of tangential C-sheets was lately analyzed by Silin and Bühner [2006] with the help of kinetic simulations which were initialized with local Maxwellian distribution functions with analytical moment profiles. Also, some evidence was found that the S-sheets could be a rotational discontinuity formed during reconnection [Gosling et al., 1982; Lyu and Kan, 1989; Lottermoser and Scholer, 1997; Dubinin et al., 2002; Lavraud et al., 2002].

Multispacecraft capabilities of the four-spacecraft Cluster mission allowed for the first time the determination of the magnetopause flapping velocity, thickness and the current density vector [Dunlop et al., 2002a, 2002b]. In this paper we utilize the capabilities of the Cluster data with the aim of showing the structure of the C- and S-sheets when they were tangential discontinuities.

Recently, Harrison and Neukirch [2009] and Neukirch et al. [2009] have shown that the preservation of the nearly constant total magnetic field across the C-sheet requires a force-free theoretical approach. They have developed a force-free model with the help of the method for finding the distribution function suggested by Channell [1976]. We further utilize Channell’s approach and develop another analytical model which describes the S-sheet.

2. Cluster Observations

Panov et al. [2008] have analyzed Cluster data of the 52 magnetopause crossings during two periods: between February 2, 2002 and June 17, 2002, and between June 1, 2003 and May 3, 2004, when the inter-spacecraft distances...
were less than 300 km. The crossing times and parameters of the magnetopause current sheets, such as normal in GSM frame of reference, the component of the magnetopause flapping velocity directed along the normal, and the magnetopause thickness can be found in Table 2 of Panov et al. [2008]. We employ data during these magnetopause crossings to further analyze the magnetopause current sheet structure. In particular, we use the high-resolution magnetic field data (22.4 or 66.7 Hz) which have been collected by the Fluxgate Magnetometer (FGM) [Balogh et al., 1997].

[10] We investigated the spacecraft data in the magnetopause frame of reference. In this frame the L-axis is pointing in the maximum variance direction of the magnetic field, and the N-axis in the minimum variance direction. The M-axis completes the LMN frame which is right-handed and orthogonal [Russell and Elphic, 1978]. The normal direction N and the flapping velocity component along the normal was determined using the multispacecraft timing technique [see Russell et al., 1983, equation (2)].

[11] Berchem and Russell [1982a] have found using ISEE spacecraft data that the rotation of the magnetic field across the current sheet may be either in form of C- or S-letter. We plotted the hodograms of the magnetic field in the current sheet plane, i.e., perpendicular to the normal direction N for the 52 magnetopause crossings by Cluster (not shown here), and found that the vast majority of the current sheets were indeed C- or S-polarized. Both polarizations were met approximately equally often.

[12] We consider below two (out of 52) magnetopause crossings by Cluster: an example of the C-sheet is given in Figure 1, and an example of the S-sheet is given in Figure 2. The two current sheets were the clearest examples in the two groups, and were also identified as tangential discontinuities (i.e., we did not find any signature of a rotational discontinuity).

[13] Figures 1 (left) and 2 (left) show three components of the magnetic field observed by Cluster-1. The other three Cluster spacecraft observed very similar current sheet structure but at a shifted time (not shown). Therefore, we can...
conclude that the current sheets were steady-stable at times needed for all four spacecraft to cross the current sheets, i.e., for at least several gyroperiods of thermal protons. Figure 1 (left) shows Cluster observations of the L, M, and N components of the magnetic field on 14 December 2003, between 06:46:10 UT and 06:46:35 UT. The length of the gradient in the Bz-component of the magnetic field along the current sheet normal direction N was on the order of 1000 km, or about seven thermal proton gyroradii. Figure 2 (left) shows Cluster observations of the L, M, and N components of the magnetic field on 6 April 2004, between 04:34:15 UT and 04:34:30 UT. The length of the gradient in the Bz-component of the magnetic field was on the order of 300 km, or about six thermal proton gyroradii.

[14] The four spacecraft magnetic field observations allowed us to calculate the magnetopause currents as curl of the current sheet normal direction, which were both positive and negative. [15] We show in section 3 that the structure of the Jz-component of the electric currents (blue curves in Figures 1 (middle) and 2 (middle)) can be explained by the gradient of the Bz-component of the magnetic field along the current sheet normal direction N (green curves in Figures 1 (left) and 2 (left)). While in Figure 1 there is only one hump in Bz, in Figure 2 one can see that the Bz-component of the magnetic field has a bipolar structure across the current sheet. This difference in the Bz behavior can also be recognized in the hodograms of the magnetic field shown in Figures 1 (right) and 2 (right). While the hodogram in Figure 1 has a shape resembling letter C, the hodogram in Figure 2 is shaped in form of letter S.

[16] The bipolar structure of the Jz-component of the electric currents (red curves in Figures 1 (middle) and 2 (middle)) suggests that it is either due to the gradient of the Bz-component of the magnetic field along the maximum variance direction L or due to a superposition of dBz/dL and dBz/dM. In this particular magnetopause crossing Jz appeared to be stronger than the Jz-component of the electric currents. Despite the obvious importance of the magnetic gradients along the tangential to the current sheet axes L and M, for the sake of simplicity we limited our discussion to magnetic gradients along the normal to the current sheet N-axis only.

3. Models of the Magnetopause Current Sheet

[17] By omitting the discussion about the magnetic field gradients along the tangential axes L and M we assume that the magnetopause current sheet has a 1D structure, i.e., all variables depend only on the z coordinate perpendicular to the magnetopause surface. The magnetic field has two components Bz(z) (equivalent of Bz in Figures 1 and 2) and Bn(z) (equivalent of BM in Figures 1 and 2). The corresponding current density components are jz(z) and jn(z), i.e., jz and jM in Figures 1 and 2.

[18] Channell [1976] proposed an approach for obtaining 1D stationary equilibria when the Vlasov-Maxwell system could be reduced to two equations for the components of the vector potential:

\[
\begin{align*}
\frac{d^2A_x}{dz^2} &= -\frac{dU}{dA_x} \\
\frac{d^2A_y}{dz^2} &= -\frac{dU}{dA_y},
\end{align*}
\]

where \( U = U(A_x, A_y) \) is the effective potential determined by particle velocity distribution functions fj (see Channell [1976] and Lee and Kan [1979] for more details):

\[
U = 4\pi \sum_j m_j \int_{-\infty}^{\infty} v^2 f_j d^3v.
\]

Here, j denotes particle species (j = i stands for ions, and j = e − for electrons). To obtain the C-sheet equilibrium with

\[ B_x \sim \tanh(z/L), \quad B_y \sim 1/\cosh(z/L), \]

we choose the vector potential components

\[ A_z \sim \arctan(\hat{z}/L), \quad A_y \sim \ln \cosh(z/L). \]

Consequently, we find that

\[
\begin{align*}
\frac{d^2A_z}{dz^2} &\sim \tanh(z/L)/\cosh(z/L) \sim \sin(2A_z) \\
\frac{d^2A_y}{dz^2} &\sim 1 - \tanh^2(z/L) \sim \exp(-2A_y),
\end{align*}
\]

and the effective potential \( U \sim \cos(2A_y) + \exp(2A_y) \). The corresponding particle velocity distribution function is

\[ fj \sim \cos(2P_y) + \exp(2P_y), \]

where \( P_x \sim A_x \) and \( P_y \sim A_y \) are components of the canonical momentum. The current sheet with such a distribution function was obtained by Harrison and Neukirch [2009]. Their model possesses the following magnetic field vector B = Bz(tanh(z/L)e_z + B0cosh−1(z/L)e_y), where B0 is the amplitude of the magnetic field and L is the current sheet thickness.

[19] For the S-sheet equilibrium with

\[ B_x \sim \tanh(z/L), \quad B_y \sim \tanh(z/L)/\cosh(z/L), \]

we chose the components of the vector potential

\[ A_z \sim 1/\cosh(z/L), \quad A_y \sim \ln\cosh(z/L). \]

Therefore, we obtain that

\[
\begin{align*}
\frac{d^2A_z}{dz^2} &\sim (1 - 2 \tanh^2(z/L))/\cosh(z/L) \\
\frac{d^2A_y}{dz^2} &\sim 1 - \tanh^2(z/L) \sim \exp(-2A_y),
\end{align*}
\]

Then one can obtain the effective potential \( U \sim A_z^2(A_z^2 - 1) + \exp(2A_y) \), and the particle velocity distribution function \( fj \sim \exp(2P_y) + P_x^2(P_x^2 - 1) \). One can search for the
distribution function as a fourth-order polynomial of $P_x$. We chose $f_j$ as a sum of two terms:

$$f_j = n_0 \left( \frac{2 \pi}{h_j^2} \right)^{3/2} \exp\left( -H_j/T_j \right) \left[ r_j \exp\left( v_{Dj} P_{Dj}/T_j \right) + r_q \exp\left( w_{Dj} P_{Dj}/T_j \right) \right].$$

Here $v_{Dj} = \sqrt{2 T_j/m_j}$ is the thermal velocity, $H_j = \frac{1}{2} m_j v_j^2 + q_j \phi$ is the total energy, $P_{Dj} = m_j v_j^2 + \frac{q_j}{e} A_j (z)$ is the momentum of particles in the direction $\alpha = x, y, v_{Dj}$ and $w_{Dj}$ are the constant drift velocities, $r_j$ and $r_q$ are free dimensionless parameters. We choose $g_j$ as a series:

$$g_j(G_{sj}) = \sum_{k=0}^{2} \dfrac{\epsilon_{sj} (2 \epsilon_{sj} + 2 G_{sj}^2)^k}{\epsilon_{sj}^k},$$

where

$$\epsilon_{sj} = w_{Dj}/v_{Dj}, G_{sj} = G_{Dj} q_j A_j/(2 T_j c),$$

and $t_{h,j}$ are free constant coefficients. From equations (5) and (6) one can find the particle density:

$$n_j = n_0 e \frac{2}{h_j^2} \left( r_j e^{2 G_{sj}^2 + r_j} + r_q D_j(G_{sj}) \right).$$

Here $G_{Dj} = \frac{q_j v_{Dj} A_j}{2 T_j c}$ and $G_{sj} = \frac{w_{Dj} q_j A_j}{2 T_j c}$. Let us define the free parameters in the following form: $r_j = e^{\epsilon_{sj} (G_{sj} - 1)}$, $r_q = 1 - r$, $t_{h,j} = -2 \epsilon_{sj} t_{h,j} - 12 \epsilon_{sj} t_{h,j} + 2 - 12 t_{h,j} e^{-\epsilon_{sj}}$. Then

$$D_j(G_{sj}) = 4 r_j G_{sj}^2 + 16 t_{h,j} G_{sj}.$$

The charge-neutrality condition can be written as (we define $q_j = -q_e$):

$$\left[ r_j e^{2 G_{sj}^2} \right] \times \left[ \epsilon_{sj} + (1-r) e^{2 G_{sj}^2} \right] D_j(G_{sj}) = \left[ r_j e^{2 G_{sj}^2} \right] \times \left[ (1-r) e^{2 G_{sj}^2} \right] D_j(G_{sj}).$$

The solution of equation (9) is:

$$\varphi = 0, \quad v_{Dj}/T_j = \frac{v_{Dj}}{T_j}, \quad w_{Dj}/T_j = \frac{w_{Dj}}{T_j}, \quad t_{h,j} = t_{h,j} = t_{h,j}.\ (10)$$

[20] The relations of the drift velocities of ions and electrons in our case are similar to the Harris current sheet model [Harris, 1962]. We introduce the dimensionless components of the vector potential: $a_i = A_i/B_0 L_y$ and $a_x = A_x/B_0 L_x$. The magnitude of the pressure field $B_0$ can be determined from the pressure balance, and with the spatial scales $L_y$ and $L_x$ found by normalizing functions $G_{oj}$:

$$\left\{ G_j = \frac{q_j v_{Dj}}{c T_j} A_j(z) = \frac{q_j v_{Dj} B_0 L_y}{2 c T_j} a_i(z) = a_i(z), \right.$$  
$$\left\{ G_j = \frac{q_j w_{Dj}}{c T_j} A_j(z) = \frac{q_j w_{Dj} B_0 L_x}{2 c T_j} a_i(z) = a_i(z). \right.\ (11)$$

From equations (10) and (11) we obtain that

$$L_y = 2 c T_j \left( q_j v_{Dj} B_0 \right) L_z = 2 c T_j \left( q_j w_{Dj} B_0 \right).$$

The particle velocity distribution functions can then be rewritten as

$$f_j = r_{fj} (1 + r^2 r_{fj}),$$

$$f_j = n_0 \left( \frac{2 \pi}{h_j^2} \right)^{3/2} \exp\left( -u_{j}^2 - u_{j}^2 - (u_j - e_i u_j)^2 \right) \left[ r_j \exp\left( v_{Dj} P_{Dj}/T_j \right) + r_q \exp\left( w_{Dj} P_{Dj}/T_j \right) \right],$$

$$f_j = n_0 \left( \frac{2 \pi}{h_j^2} \right)^{3/2} \exp\left( -u_{j}^2 - u_{j}^2 - (u_j - e_i u_j)^2 \right) \left[ t_{h,j} (2 e_i u_j + 2 a_x(z)) \right].$$

Here $u_j = v/v_{Dj}$. The corresponding current density components can be obtained by integrating equations (12):

$$j_y = n_0 v_{Dj} \exp(2 a_x(z)),\ \left. \right.$$  
$$\left. j_x = (1-r) n_0 w_{Dj} \right. \sum_{k=0}^{2} t_{h,j} (2 e_i u_j + 2 a_x(z)).\ (13)$$

where $\tau = T_j/T_e$. We also set $t_2 = 1/4$, and $t_4 = -1/16$. The Maxwell’s equations for the vector potential have the following form:

$$\frac{d^2 a_x}{d z^2} = -\frac{4 \pi}{c B_0} q_0 n_0 v_{Dj} L_y \left( 1 + \tau \right) e^{2 a_x},$$

$$\frac{d^2 a_y}{d z^2} = -\frac{4 \pi}{c B_0} q_0 w_{Dj} L_x \left( 1 + \tau \right) a_x(1 - 2a_x^2).$$

[21] Here we introduced the dimensionless spatial variables $\zeta_y = \sqrt{r z/L_y}$ and $\zeta_x = \sqrt{1 - r z/L_x}$. By choosing the magnitude of the magnetic field $B_0 = 8 \pi (T_j + T_e) n_0$ we now obtain the solution to system (14) (here we assume that $B_x$ and $B_y$ are vanishing at the neutral plane $z = 0$):

$$\left\{ a_y = \frac{1}{2} \ln \cosh (\zeta_y), \right.$$  
$$\left. a_x = \cosh^{-1} (\zeta_x). \right.\ (15)$$

By choosing the relation between $w_{Dj}$ and $v_{Dj}$ one can find that $L = \sqrt{r L_y} = \sqrt{1 - r L_x}$, and finally come to the S-sheet equilibrium:

$$B_y/B_0 = \tanh (z/L),$$

$$B_y/B_0 = -\tanh (z/L) \cosh^{-1} (z/L),$$

$$4 \pi j_y/c = (B_0/L) (1 - 2 \tanh^2 (z/L)) \cosh^{-1} (z/L),$$

$$4 \pi j_y/c = (B_0/L) \cosh^{-2} (z/L).$$

[22] We show the magnetic field and current density components for the C- and S-sheets in Figure 3. One can see that they resemble reasonably well the spacecraft observations shown in Figures 1 and 2. While the C-sheet has an odd bipolar $J_y$, the S-sheet possesses an even $J_y$ with stronger positive currents in the center and two weaker negative currents at the periphery of the current sheet.

[23] The total magnetic field pressure $(B_y^2 + B_z^2)/B_0^2$ as a function of the normal $z$-coordinate is shown in Figure 4. While the C-sheet solution is a force-free equilibrium with $B_y^2 + B_z^2 = \text{const}$, the S-sheet solution has a minimum in the magnetic pressure profile at the center of the current sheet. Due to the pressure balance in the sheets the normalized plasma pressure $8 \pi P_{zz}/B_0^2$ could be found as a difference $8 \pi P_{zz}/B_0^2 = \text{const} - (B_y^2 + B_z^2)/B_0^2$. For the C-sheet $8 \pi P_{zz}/B_0^2$ is a constant determined by the particle temperatures and densities, see Neukirch et al. [2009]. In the case of the S-sheet, $8 \pi P_{zz}/B_0^2$ has a maximum in the central region and decreases toward the boundaries of the current sheet (see Figure 4, right).

[24] In the developed S-sheet model (as well as in C-sheet model) particle temperature is a constant $(T_j = \text{const})$. Therefore, the plasma pressure across the S-sheet is proportional to the plasma density. However, simple manipulations with the particle velocity distribution function $f_j$ can provide us with the S-sheet equilibrium, where variations of the plasma pressure are defined by variations in the temperature.
only, or by a superposition of variations in the plasma temperature and density (see Appendix A for details).

4. Conclusions and Discussion

[25] From the previous observations it was well known that the rotation of the magnetic field across the magnetopause current sheet can be either in form of C or S letters [e.g., Berchem and Russell, 1982a]. In this paper we have analyzed tangential C- and S-sheets observed by the four closely spaced Cluster spacecraft while crossing the Earth’s magnetopause.

[26] We have demonstrated the current sheet structure for both the C- and S-sheets using the multispacecraft Curlometer technique [Dunlop et al., 2002b]. We have shown that the structure of the C-type tangential current sheet can be described analytically following Harrison and Neukirch [2009], with the help of the method for finding the

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Figure 3. Profiles of (left) the magnetic fields, (middle) the current densities, and (right) the magnetic field hodographs for (top) the C-type sheets and (bottom) the S-type sheets.

Figure 4. Profiles of the magnetic pressure components $B_x^2$, $B_y^2$, and $B_x^2 + B_y^2$ for (left) the C-sheet and (right) the S-sheet. For the S-sheet the plasma pressure profile $8\pi P_{zz}/B_0^2$ is also plotted.
distribution function suggested by Channell [1976]. Although, it was suggested that the S-sheets could be a rotational discontinuity formed during reconnection [Gosling et al., 1982; Lyu and Kan, 1989; Lottermoser and Scholer, 1997; Dubinin et al., 2002; Lavraud et al., 2002], we have developed another analytical equilibrium with the help of Channell’s approach which describes tangential S-sheet. In addition to the demonstrated kinetic C- and S-sheet equilibria, both current sheet types were also obtained in the magnetohydrodynamic (MHD) approximation (see Appendix B for details).

Appendix A: Equilibria With Nonuniform Temperature

[34] The kinetic equilibria for the C- and S-sheet were obtained using the following particle velocity distribution functions:

\[ f_j(H_j, P_{jx}, P_{jy}, T_j, n_0) = n_0 C_j e^{-\frac{H_j}{T_j}} g(P_{jx}, P_{jy}), \]

where \( C_j \) is a normalizing constant and \( g \) is a function of particle momentum. In this appendix we omit variables \( H_j \), \( P_{jx}, P_{jy} \) and write the particle velocity distribution function as a function of temperature and density only \( f_j = f_j(T_j, n_0) \). One can write the first three moments of \( f_j \) as

\[ n_j = \int_{-\infty}^{+\infty} f_j d^3 v = n_0 N(z) \]

\[ j_j = q_j \int_{-\infty}^{+\infty} v f_j d^3 v = q_j n_0 \sqrt{2 T_j/m_j J(z)} \]

where \( J(z) \) is a function of the z-coordinate defined by the dependence of \( f_j \) on \( H_j \), \( P_{jx}, P_{jy} \). The particle temperature for this system does not depend on the z-coordinate: \( T_j = P_{z,j}/n_j = T_j = \text{const} \). Therefore, the plasma pressure \( P_{z,j} \) is proportional to the particle density \( n_j \).

[35] In order to allow a non-uniform \( T_j = T_j(z) \), one can construct a particle velocity distribution function which is a sum three distribution functions: two distribution functions \( f_{1j} \) and \( f_{2j} \) in the form of \( f_j \), but with different temperatures, i.e., \( f_{1j} = f_j(T_{1j}, n_0) \), and another particle velocity distribution function of a background plasma with homogeneous density \( f_{bg,j} \):

\[ f_j = f_{bg,j} + f_{1j} + f_{2j} \]

\[ \lambda = \text{const} \]

\[ f_{bg,j} = f_{bg,j}(T_{bg,j}, n_{bg}) = n_{bg} C_{bg,j} \exp(-H_j/T_{bg,j}) \]

If we assume that \( T_{1j} \) is greater than \( T_{bg,j} \), one can write the first three moments of \( f_j \) as:

\[ n_j = \int_{-\infty}^{+\infty} (f_{bg,j} + f_{1j} + f_{2j}) d^3 v = n_{bg} + (1 + \lambda)n_0 N(z) \]

\[ j_j = q_j \int_{-\infty}^{+\infty} v (f_{bg,j} + f_{1j} + f_{2j}) d^3 v = q_j n_0 \left( \sqrt{\frac{2 T_j}{m_j}} + \lambda \sqrt{\frac{2 T_{bg,j}}{m_j}} \right) J(z). \]

\[ P_{z,j} = \int_{-\infty}^{+\infty} v^2 m_j f_j d^3 v = n_{bg} T_{bg,j} + (T_j + \lambda T_{1j}) n_0 N(z) \]
For $\lambda = 0$ and $n_{bg} = 0$ we will obtain system (A2). For $\lambda = -1$ we have a constant density $n_1 = n_{bg}$ and nonuniform temperature $T_{\text{eff}} = T_{bg,j} + (T_{ij} - T_{ij}) \frac{n_2}{n_{bg}} N(z)$. In this case the plasma pressure is proportional to temperature. By varying parameter $l$ in the range between $-1$ and 0, one can obtain a class of density and temperature profiles for the same profile of the plasma pressure components $P_{\text{eff}}$. For any $l \in [-1, 0]$ the condition $T_{ij} > T_{ij}$ provides the conservation of the current density profile and the corresponding profiles of the magnetic field.

Appendix B: MHD C- and S-Sheet Equilibria

[37] In this appendix we develop magnetohydrodynamic (MHD) equilibria for the C- and S-sheets. For this purpose we consider a stationary single-fluid system after Shkarofsky et al. [1966]:

\[
\begin{aligned}
\nabla P &= \frac{1}{c} [j \times B] \\
\nabla \times B &= \frac{\partial T}{c} j \\
\n\nabla B &= 0
\end{aligned}
\]  

(B1)

By introducing the vector potential $[\nabla \times A] = B$, one can rewrite the second equation of system (B1): $c^2 \frac{\partial^2 A}{\partial x^2} = \frac{4 \pi j}{c}$. We assume, that the plasma pressure is a function of the vector potential $P = P(A)$. Then, the pressure tensor gradient can be written as

\[
\nabla P = (\frac{\partial P}{\partial z}) e_z = (\frac{\partial P}{\partial A_x}) B_x e_x - (\frac{\partial P}{\partial A_y}) B_y e_y.
\]

(B2)

Therefore, system (B1) can be rewritten as

\[
\begin{aligned}
\frac{\partial^2 A_x}{\partial x^2} &= -4\pi \frac{\partial P}{\partial A_x} \\
\frac{\partial^2 A_y}{\partial x^2} &= -4\pi \frac{\partial P}{\partial A_y}
\end{aligned}
\]

(B3)

[38] This system is equivalent to system (1) where $U = 4\pi P$. Following the results by Birn et al. [1975] and Hilmer and Voigt [1987] we define $P$ as

\[
P = P_0 (A_1/A_0)^2 \left(1 - (A_0/A_0)^2 \right) + P_0 e^{2(A_0/A_0)},
\]

(B4)

where $A_0 = B_0 L$. Now we obtain expressions for the dimensionless components of the vector potential:

\[
\begin{aligned}
\frac{\partial^2 a_x}{\partial c^2} &= -\frac{8\pi P_0}{B_0^2} a_x (1 - 2a_c) \\
\frac{\partial^2 a_y}{\partial c^2} &= -\frac{8\pi P_0}{B_0^2} e^{a_c}
\end{aligned}
\]

(B5)

If we assume that $B_0^2 = 8\pi P_0$, one can obtain solutions for $a_0(\xi)$, $a_1(\xi)$ similar to equations (15). This way, one can develop the S-sheet equilibrium in MHD approximation.

The C-sheet equilibrium can be obtained similarly, and assuming that $P = P_0 \cos(2\xi) + P_0 e^{2\xi}$.

[39] Acknowledgments. The authors highly appreciate the work of the Cluster FGM team and the Cluster Active Archive for the processing of the magnetic field data. The work of A.V.A. was supported by the Russian Federation Presidential Program for State Support of Leading Scientific Projects (NSH-3200.2010.2) and by the RFBR project 10-02-93114. The work was partly supported by the Austrian Science Fund (FWF) 1429-N16 and by the Seventh Framework Programme (FP7, project 269198, “Geoplasmas”) from the European Commission. The authors greatly acknowledge two reviewers for useful suggestions and interesting discussions which encouraged the improvement of the manuscript.

[40] Philippa Browning thanks the reviewers for their assistance in evaluating this paper.

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