The role of the Hall effect in collisionless magnetic reconnection

R.A. Treumann a,c,*, C.H. Jaroschek b,a,d, R. Nakamura e, A. Runov e, M. Scholer a

a Max-Planck-Institute for extraterrestrial Physics, Karl-Schwarzschildstr. 1, D-85748 Garching, Germany
b Universitätsternwarte, Ludwig-Maximilians-Universität, München, Germany
c Department of Geosciences, Ludwig-Maximilians-Universität, Munich, Germany
d International Max-Planck-Research-School on Astrophysics, MPG and LMU Munich, Germany
e Institute of Space Research, Austrian Academy of Sciences, Graz, Austria

Received 20 August 2004; received in revised form 11 November 2004; accepted 11 November 2004

Abstract

A short review of the role of the Hall effect in collisionless reconnection is given in relation to observations in the upper auroral ionosphere during substorm-reconnection events. We clarify the conditions when the Hall effect should be of importance and when the ion inertial length is the responsible scale which is the case for $\beta_i > 1$. Moreover, we relate the auroral acceleration region to the reconnection region in the tail determining the relations between upward and downward currents and the different ranges of the Hall reconnection region in the tail. It is concluded that the downward–upward currents can be mapped to those domains in the near-Earth tail and suggest that these regions are in fact very narrow and that due to this narrowness and the abundance of upward–downward current regions reconnection in near-Earth space during substorms is probably highly patchy.

© 2006 Published by Elsevier Ltd on behalf of COSPAR.

Keywords: Hall effect; Collisionless reconnection; Field-aligned currents; Aurora

1. Introduction

Collisionless reconnection is the form of reconnection encountered in the dilute, collisionless magnetized plasmas in near-Earth space: at the magnetopause and in the neutral sheet of the tail. Collisionless reconnection is one of the most important and most fundamental energy release processes in collisionless plasmas. Reconnection has originally been proposed by Giovanelli (1947, 1948) as a collisional mechanism possibly responsible for converting stored magnetic energy into kinetic particle energy. It has been elaborated by Sweet (1958) and Parker (1957, 1963) in the context of colliding magnetic fields in the solar atmosphere during solar flares, today a widely accepted though still not confirmed nor understood mechanism for solar flare energy release. Under collisional conditions reconnection is slow (see, e.g., Biskamp, 2000), limited by the product of Alfvén and resistive times, $\sqrt{\tau_A \tau_R}$. It assumes that the spatial extent of the reconnection site, the so-called diffusion region where the annihilation of the magnetic fields takes place, is much larger than the mean free path of the plasma particles involved. This assumption makes collisional reconnection accessible to a fluid-like treatment either in MHD or in a two- or multi-fluid plasma model. Collisionless reconnection, on the other hand, happens on a time scale that is vastly shorter than any binary collisional or resistive scale. It can thus be very fast, much faster indeed than the above time scales allow. It also takes place on spatial scales much shorter than any mean free path of the plasma particles.

Reconnection involves current sheets of thickness $d$. The physics of such current sheets in ordinary MHD has been investigated in detail by Syrovatskii (1981). Assuming that any arbitrary component of the plasma can be treated as a
fluid implicitly implies that this component is collisional. Some thermalization mechanism has been at work such that the particles do not behave any more as particles but collectively form fluid elements. In this sense hybrid models which treat the ions as particles while the electrons are assumed a reacting fluid, which in most cases is taken as a Boltzmann fluid, are not collisionless.

For reconnection to be collisionless the mean free path of the particles, both electrons and ions, must exceed the typical extension of the diffusion region. The type of diffusion at the reconnection site which is responsible for the separation between the motion of the plasma particles and the magnetic field is not yet precisely known. In order to separate the plasma from the magnetic field and break the frozen-in condition (which holds strictly only in ideal MHD but may still be strong enough to retard reconnection to unreasonably long times) requires that the particle component becomes unmagnetized. This requires that the cyclotron radius of mass and charge \( m \) and \( e \) the particle of the particle mass and charge \( e \) in the field \( B \) at the inflow (lobe) edge of the diffusion region.

A reasonable assumption is that the diffusion region is at least as large in size as the ion inertial length \( \lambda_i = c/\omega_{ci} \). This assumption is not arbitrary as an ion with \( r_{ci} > \lambda_i \) will on this scale follow its inertia and when being exposed to an electric field will be accelerated in this field. Writing this condition (for thermal particles, i.e., for the main particle component) as

\[
nkBT > \frac{B^2}{2\mu_0},
\]

it is seen that the condition for non-magnetization of the (thermal) plasma on the ion inertial length is trivially that \( \beta_i > 1 \). Applied to reconnection this implies that in a high-\( \beta \) plasma with the mean free path being larger than the ion cyclotron radius, any current layer of extent \( d \lesssim \lambda_i \) will behave completely collisionless. PIC simulations in 2D and 3D show that it also will undergo spontaneous reconnection.

This argument can be strengthened by comparing the mean free path with either the cyclotron radius of an ion or the ion inertial length. The mean free path is given by \( \lambda_{\text{mfp}} = \nu_{\text{mfp}} / v_r \) with \( v_r \) the Spitzer collision frequency based on Coulomb interaction. For \( r_{ci} > \lambda_{\text{mfp}} \) we then obtain that

\[
N_D \equiv \frac{2\pi}{3} nkT \lambda_D^3 > \frac{3}{c} \frac{v_A}{v},
\]

where \( \lambda_D \) is the Debye length and \( v_A \) the Alfvén speed. The left-hand side is the number of particles in a Debye sphere which for a plasma is required to be very large, \( N_D \gg 1 \), usually \( \log N_D \sim 15-20 \) such that this condition is almost always satisfied. This results again in the rather trivial conclusion that a \( \beta_i > 1 \) plasma will almost always be collisionless on the ion inertial scale and that thin current sheets \( d \lesssim \lambda_i \) are collisionless.

The mean free path \( \lambda_{\text{mfp}} \) is related to the resistive length scale, which is defined setting the magnetic Reynolds number \( R_m = \mu_0 LV / \eta \). This yields \( L_m = \eta / (\mu_0 V) \), where \( \eta = v_c / \epsilon_0 c \) is the resistivity. Inserting for \( v_c \) and using the above condition on \( N_D \) gives a condition on the resistive scale length in terms of the ion inertial length and the ratio of the Alfvén speed to the characteristic macroscopic velocity \( V \) as

\[
\frac{L_m}{\lambda_i} < \frac{\bar{m}}{\nu} \ln N_D \frac{v_A}{V}.
\]

In order for resistive reconnection to occur one requires \( \lambda_i \sim \lambda_i \) and hence

\[
\frac{V}{v_A} \lesssim \frac{1.5}{\ln N_D} \frac{\bar{m}}{\mu_0} \approx 500,
\]

a condition which is neither satisfied anywhere in the magnetosphere nor the solar wind. Even in the solar corona it will be satisfied only for weak fields and dense plasma conditions in the presence of fast streams. One may thus conclude that resistive reconnection will not occur anywhere in these systems.

Resistivity may, however, be localized and anomalous, generated by short wavelength plasma waves which scatter the particles. In this case the collision frequency becomes

\[
\nu_{\text{an}} \approx \omega_{pe} \frac{W}{nk_BT},
\]

where \( W \) is the (integrated over the spectrum) wave energy density. The plasma will then be collisionless whenever

\[
\frac{v_A}{c} > \sqrt{\frac{\bar{m}}{\mu_0}} \frac{W}{nk_BT} = \sqrt{\frac{\bar{m}}{\mu_0}} \frac{W}{B^2 / 2\mu_0} \beta^{-1}.
\]

The first part of this equation holds for electrostatic, the second part for electromagnetic fluctuations. A canonical number for electromagnetic fluctuations is \( W/nk_BT \sim 10^{-5} \). Hence, the condition for being collisionless is

\[
\frac{v_A}{c} > 10^{-4}.
\]

However, current sheets must in addition satisfy the condition \( \beta > 1 \). Hence, rewriting the last expression yields

\[
\frac{k_BT}{\bar{m}c^2} > 10^{-8} \beta,
\]

which says that plasmas of ion temperature \( k_BT \gg 10 \) eV are collisionless on the ion inertial scale with respect to anomalous resistivities generated by electrostatic fluctuations. This condition does, however, not exclude very strong localized fluctuations like electron holes which may produce a strongly localized anomalous resistivity. If this is not the case, any thin current sheets will behave collisionless within an ion inertial length.

In the following we briefly investigate the processes taking place within the collisionless ion inertial scale. We will
exclude the electron inertial scale from consideration and restrict to the discussion of the plasma dynamics under the assumption that the diffusion causing reconnection proceeds entirely inside the ion inertial length. Processes taking place on the electron inertial scale when the current layer electrons become unmagnetized may be of crucial importance for reconnection but fall outside the investigation of this paper.

2. Physics on the ion inertial scale

Let us assume that the initial two-dimensional current sheet separating the antiparallel fields like in the tail of the magnetosphere is sufficiently thin with diameter \( d \ll \lambda_i \), and \( \beta_\perp > 1 \). In this case the ions are unmagnetized inside and whenever they cross the current layer. Since the electrons are magnetized, and the plasma is streaming in from the magnetospheric lobes at the magnetospheric convection velocity, the electrons are tied to the decreasing magnetic field moving across the ion component at increasing inflow velocity \( u_z = E_y/B_x \). Here \( E_y \) is the lobe convection electric field. This is the usual scenario of two-dimensional reconnection which is schematically shown in Fig. 1.

Sonnerup (1979) was the first to realize the importance of this difference between the electron and ion motion which leads to a current flowing antiparallel to the inward convection of the electrons and perpendicular to both the convection electric and the lobe magnetic fields. It hence represents a true Hall current in the collisionless plasma. This Hall current is defined as flowing strictly transverse to the electric and magnetic fields. It does not contain a component parallel to the magnetic field. Hence, it either closes in itself as in the equivalent currents in the ionosphere, or its divergence has to be closed by field-aligned currents. In fact, the Hall currents in the ion inertial region are not free of divergence. They therefore become a source of magnetic field-aligned current flow.

There is a number of important facts to note about the Hall current scenario shown in Fig. 1:

- the representation in Fig. 1 is entirely two-dimensional; in first approximation the presence of the Hall current does not change the geometry into three dimensions; the variation of the Hall magnetic field is still restricted only to the \((x,z)\)-plane;
- however, the Hall current is an indication of two facts: the collisionless nature of the plasma inside the ion inertial domain (possibly with the exclusion of the electron inertial region), and the presence of reconnection in the current layer;

![Fig. 1. The Hall current system schematics in collisionless reconnection in thin current sheets. The large circle is the ion inertial domain of radius \( \lambda_i \). The small inner circle in light blue is the electron inertial region. Red arrows indicate the slow convective inflow \( u_{\text{conv}} \) of electrons and the fast (close to Alfvén speed) reconnection jet outflow \( u_{\text{rec}} \) velocities. The thin blue arrows opposite to the electron flows are the Hall currents which generate the quadrupolar Hall magnetic field components \( B_{y\text{Hall}} \). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)](image-url)
• the Hall current is non-zero only in the ion inertial region close to but outside the X-line which is the crossing point of the two separatrices;
• by definition, electron Hall currents \( j_H = -enE \times B/b^2 \) are perpendicular to the magnetic and electric fields; they vanish on the boundaries of the ion inertial region. In contrast to Hall currents in a resistive medium like the ionosphere the Hall currents here are not free of divergence;
• the divergence of the Hall currents is the source of field-aligned current flow from and into the ion inertial region;
• the Hall magnetic field component is quadrupolar and points either in positive or negative \( y \) direction; there is no \( z \) component of the Hall field; therefore the Hall field does not contribute to reconnection;
• the Hall magnetic field is a secondary guide field generated by the difference between the ion and electron flows;
• this guide field vanishes in the center of the current sheet and exists merely at its boundaries;
• the Hall magnetic field is parallel (antiparallel) to the convection electric field; it thus enables moderate acceleration (retardation) of particles in the boundary regions of the current sheet thereby increasing the cross-tail current; this causes flattening of the current profile or even a bifurcation of the current.

From these items one can draw a number of conclusions which lead to a physical model of the plasma dynamics in the ion inertial range and its connection to auroral physics.

The above scenario builds on quasi-stationary reconnection, it suggests that reconnection is in equilibrium. There is, however, no reason for such a stationary state to exist as it requires a subtle balance between the inflow of plasma and magnetic field into the ion inertial range, the annihilation of magnetic fields, and the accelerated outflow of plasma from the reconnection site. Simulations (see, e.g., Scholer et al., 2003; Jaroschek et al., 2004) show that when no new plasma is supplied, the X-line region stretches in the outflow direction and the plasma around the X-line thins out. Hence, when the inflow is too slow fast collisionless reconnection proceeds on its own explosive scale, disrupts the current, ejects plasma from the X-line, and subsequently relaxes until sufficient magnetic field and plasma has accumulated again in the current sheet to ignite new reconnection. On the other hand, when the inflow is very strong its main effect is that the current sheet widens. Reconnection in this case is not fast enough of being in the position to account for the plasma inflow. The widths of the current sheet increases until \( d > \lambda_i \) becomes larger than the ion inertial length. Reconnection then ceases to be fast. In fact it becomes slow, the case investigated by Syrovatskii (1981). Numerical simulations show that such broad current sheets are stable with respect to fast reconnection and, moreover, are not subject to the Hall effect. Thus the importance of the Hall effect is restricted to thin current sheets of width \( d \ll \lambda_i \). In order to have fast reconnection in the presence of strong plasma inflow the current sheet must be kept narrow by some mechanism like compression. However, reconnection can still become fast even in this case whenever the current sheet manages to bifurcate (Runov et al., 2003a,b) into many narrow current filaments or sheets of widths \( d < \lambda_i \).

From this point of view it is not the Hall term which is necessary for fast reconnection to occur but the narrowness of the current sheet. The Hall effect is a secondary effect which arises whenever current sheets become thin enough for the ion and electron dynamics to decouple. In fact, PIC simulations of pair plasmas (Zenitani and Hoshino, 2001; Jaroschek et al., 2004) where the Hall effect is entirely absent on all scales support this conclusion as they show that fast reconnection takes place spontaneously whenever the current sheets become thinner than the particle skin length.

The Hall effect itself fails in driving reconnection because it does not generate a \( z \)-component of the magnetic field near the X-line. It also fails in another respect: it does not produce dissipation. For the latter one requires that the product \( jE \neq 0 \) be different from zero. However, for the Hall current \( j_H E_{\text{conv}} = 0 \) vanishes identically. Since no (direct) dissipation is produced by the Hall current it is immediately clear that Hall currents cannot be directly involved in reconnection as it is instrumental for reconnection to base itself on whatever kind of dissipation. The Hall effect, however, may contribute to other non-dissipative effects which secondarily could cause dissipation.

Because of these properties it has been proposed that in thin current layers the Hall effect secondarily leads to the generation of large amplitude nonlinear whistlers (Drake et al., 1994; Mandt et al., 1994; Rogers et al., 2003) standing in the current sheet. Since such whistlers propagate parallel to the magnetic field while being transverse electromagnetic waves they naturally possess a magnetic \( z \)-component that may cause reconnection. Hall-MHD and electron-MHD simulations seem indeed to support this proposal.

The Hall effect in the ion inertial region generates a weak out-of-plane quadrupolar field which plays the role of a weak guide field. Such guide fields have the general negative effect of retarding reconnection (Scholer et al., 2003; Pritchett and Coroniti, 2004). In addition strong guide fields support the formation of electron holes (Drake et al., 2003) which cause additional acceleration of particles and contribute to the three-dimensional evolution of reconnection. The condition for electron holes to evolve is that the plasma becomes Buneman unstable. This happens when the bulk plasma current drift velocity \( u_B > v_{\text{eth}} \) exceeds the electron thermal velocity. The Hall current speed, which is the inflow velocity of the convective flow, is far below this limit such that the Hall currents themselves cannot become Buneman unstable. However, in the presence of a guide field along the cross-tail electric field, electrons become accelerated along the field and may reach...
velocities large enough for the Buneman instability to occur. The Hall magnetic fields have this property as they point in y direction parallel (or antiparallel) to the cross-tail electric field. One therefore expects that the Hall field regions sometimes contain localized electric field fluctuations along the magnetic Hall component.

The interesting effect of such an acceleration is that the Hall field region becomes a region of amplified cross-tail current. Since the Hall field is displaced from the center of the main current flow, a quadrupolar current structure is generated which appears as a bifurcated current flow. This effect has been measured both near the magnetopause (Mozer et al., 2002 for a detailed description see the caption of Fig. 2) and in the tail plasma sheet (Runov et al., 2003a) during Hall reconnection. In tail reconnection the effect becomes much more pronounced due to the symmetrical conditions in the tail and the much larger system. For the full extended discussion of these measurements we refer to the original paper of Runov et al. (2003a).

The measurements near the magnetopause are given in overview in Fig. 2 and have been analyzed by Bale et al. (2002) as shown in Fig. 3. The intriguing observation applying to our discussion is the double step profile of the main magnetic component when crossing the reconnection site. For an ideal non-Hall Harris sheet this profile should be smooth enabling one to fit it with a single Harris profile. This has been done by Bale et al. (2002) and is shown by the dotted line which should produce the gaussian-shaped dotted current profile in the second from bottom panel in this figure. Instead, the measured current is top-flat. This results from the double-Harris profile included as the second blue fit to the upper part of the profile. The current is hence reduced in the center and spread to both sides consisting in fact of two Harris-current sheets. This effect is what is expected for a Hall-dominated collisionless reconnection region. Mozer et al. (2002) have shown that their Polar crossing was indeed Hall dominated.

Fig. 2. The Hall current system as observed in a Polar magnetopause crossing (Mozer et al., 2002). From top to bottom is shown the plasma density, the magnetic field magnitude and the components of the magnetic field, and the components of the electric field in a frame fixed to the magnetopause and in which the plasma is incident in a direction normal to the magnetopause surface. The crossing exhibits a typical Harris layer main magnetic field profile $B_z$ and a weaker sinusoidal Hall field profile $B_y$. As the normal magnetic field $B_x$ is nearly constant and close to zero, the current structure is close to two-dimensional. The density exhibits two minima (dips) at the outer boundaries of the Hall regions. The origin of the bipolar electric field component $E_x$ is not known. When combined with the magnetic field it suggests that the plasma to both sides of the magnetopause in this particular crossing is moving to south–east.
A weak indication of a similar effect is found in full partic-
le simulations (Scholer et al., 2003). Fig. 4 in the top panel
shows the electron current density measured in these simula-
tions. It exhibits an amplification in the center of the current
layer indicating electron acceleration. It also exhibits two
adjacent regions of strongly weakened current density and
at the boundaries of the initial current layer the indication
of two secondary bifurcated electron current filaments.
The mass ratio in these simulations was $m_i/m_e = 150$, still
unrealistically small such that the Hall effect will only be par-
tially expressed, however. Therefore the simulation result
may still be inconclusive in this respect. Nevertheless, the
amplification in the center of the current sheet is due to par-
ticle acceleration and subsequent current contraction in the
center. The secondary signatures are at the positions of the
maxima of the Hall magnetic fields (see Fig. 6).

An important question is whether or not the Hall effect
contributes to particle acceleration. Inside the ion-diffusion
region ions are unmagnetized. They see the convection elec-
tric field of the Hall electrons and can pick up energy. This
field is of the order of $E \sim 10^{-3}$ V m$^{-1}$. For a long exten-
sion of the X-line in y direction one must consider that
the ions are magnetized in this direction by the $B_z$ magnetic reconnection field. In the tail this is of the order of $B_z \approx 1$ nT. Ions feel the electric field during half a cyclotron period or $\Delta t_{ci} \approx 10$ s. In this time they gain a velocity increase of $\Delta v \approx 10^6 \text{ m/s}$ corresponding to an energy increase of $\sim 10 \text{ keV}$, which is substantial but still much less than the energies up to $300 \text{ keV}$ which have been reported (Øieroset et al., 2002) from observation. The main acceleration is therefore not provided by the Hall effect but by a different process which has been identified as the action of the inductive electric field generated in the reconnection (Jaroschek et al., 2004). Electron acceleration along the Hall guide field is even weaker since the electrons are magnetized. They are heated by lower hybrid waves evolving in the steep density gradients at the edges of the current sheet and sufficiently low $\beta$. Such waves have been observed (see Fig. 4) both in space (Bale et al., 2002; Vaivads et al., 2004) and in full particle reconnection simulations (Shinohara et al., 2001; Scholer et al., 2003). The dominant acceleration of electrons takes place when they enter the electron inertial scale region around the X-line where they become accelerated by the induction electric field. For non-Hall reconnection Jaroschek et al. (2004) found acceleration in the induction electric field of reconnection generates a power law distribution in the electrons. This process will dominate the electron dynamics also in the presence of ions and the Hall effect. The observations show (Øieroset et al., 2002) that the energies reached are high enough for the currents to satisfy the conditions for onset of Buneman instability and generation of electron holes. Recent observations near the magnetopause of electric fluctuations in crossings of the Hall region in reconnection (Vaivads et al., 2004; Khotyaintsev et al., 2004) confirm the existence of both electron waves and large amplitude electric structures.

3. Particle fluxes and field-aligned currents

Fujimoto et al. (1997) first observed electrons streaming in and out of the reconnection region in the tail. These observations have been confirmed and refined by subsequent Geotail (Nagai et al., 2001, 2002, 2003) and Wind (Øieroset et al., 2001) observations. Most of these measurements were made during crossings of the Hall region in tail reconnection. The most interesting fact about these observations was that energetic electrons were observed to stream out of the reconnection site along the magnetic field and at the same time low energy electrons were seen flowing into the reconnection region as well along the magnetic field. One is not talking about the inflow lobe electrons here which are transported into the reconnection region from the lobes across the field (or, strictly speaking, with the magnetic field) by convection.

If these flows are not compensated by ions flowing in the same direction they indicate that the Hall region in reconnection is the source of field-aligned currents. This is as well suggested by the apparent divergence of the Hall currents. Such currents were already included in the original treatment of the Hall current by Sonnerup (1979). However, since Hall currents are by definition exactly perpendicular to the magnetic and electric fields, respectively, the field-aligned currents resulting from the ion inertial range are strictly speaking no Hall currents. They arise merely due to the spatial bounds of the ion inertial range as seen in Fig. 1.

A simple expression for the field-aligned currents resulting from the spatially restricted ion-diffusion Hall current region can be given as follows. Taking the divergence of the Hall current $j = -enE_{\text{conv}} \times B/B^2$ yields

$$\nabla \cdot j_H = -e(nE_{\text{conv}} \times B) \cdot \nabla \left( \frac{n}{B^2} \right) - e \frac{n}{B^2} \nabla \cdot (E_{\text{conv}} \times B).$$

(9)
where the electrons transport the current at their convective drift speed, and \( n \) is the density. The vector product in the second term in this equation can be written as
\[
\frac{\mathbf{r}}{C_1} \left( \mathbf{E}_{\text{conv}} / C_2 \right) \mathbf{B} = \frac{\mathbf{B}}{C_1} \left( \frac{\mathbf{r}}{C_2} \mathbf{E}_{\text{conv}} \right) / C_0 \mathbf{E}_{\text{conv}} / C_1 \left( \frac{\mathbf{r}}{C_2} \mathbf{B} \right).
\]
\[ (10) \]

Under stationary conditions the first of these terms on the right vanishes. In the second term the Hall current contribution to \( \nabla \times \mathbf{B} = \mu_0 \mathbf{j} \) vanishes because it is perpendicular to the convection electric field. However, the cross-tail current will contribute the non-Hall term \( -\mu_0 \mathbf{E}_{\text{conv}} \mathbf{j}_{\text{tail}} \). One thus finds that
\[
\nabla \cdot \mathbf{j}_H = -e (\mathbf{E}_{\text{conv}} \times \mathbf{B}) \cdot \nabla \left( \frac{n}{B^2} \right) + \frac{\mu_0 e n}{B^2} \mathbf{E}_{\text{conv}} \cdot \mathbf{j}_{\text{tail}}.
\]
\[ (11) \]

The last term is positive as the convection field and cross-tail currents are parallel. The dissipation of the cross-tail current contributes directly to the divergence of the Hall current and thus as well to the generation of field-aligned currents. Since both, the density and magnetic field change across the ion-diffusion region and the ion-diffusion region is limited in space this divergence is non-zero and has to be compensated by the field-aligned derivative of the field-aligned current \( \nabla_\parallel \mathbf{j}_\parallel \) which, when integrating over the
field-aligned extension $L_{||}$ of the ion-diffusion region yields for the locally generated field-aligned current emanating from the ion-diffusion region

$$j_{||}(x,z) = e \int_{L_{||}} \left\{ (E_{\text{conv}} \times B) \cdot \nabla \left( \frac{n}{B^2} \right) - \frac{\mu_0 e n}{B^2} E_{\text{conv}} \cdot \mathbf{j}_{\text{tail}} \right\} ds.$$  \hspace{1cm} (12)

Here $s$ is the coordinate along the magnetic field. Of course it is difficult to solve this equation as the extension $L_{||}$ is barely known. However, this expression shows that each point along the electron trajectories across the Hall region is either a source or a sink of a field-aligned current, depending on the sign of the integrand.

The disappearance of the Hall current on the lobe side of the ion inertial range implies that at this location either ions must flow out of the ion inertial range along the magnetic field or electrons must flow into the ion inertial range or both in order to provide the closing field-aligned current. On the other hand, at the earthward boundary of the ion inertial range electrons must flow out of the inertial range region toward the earth along the magnetic field or ion must flow in for providing the closure field-aligned current. At this latter location we have no problem with the answer. The outflowing electrons which constitute the (upward) field-aligned currents flowing into the ion inertial region are the original lobe electrons which have been accelerated during passage of the ion inertial range and in the reconnection process.

The problem is slightly more difficult at the lobe boundary of the ion inertial range region. The observation of low energy electrons flowing in at this place is a strong argument that here the source is suspected to be in the upper ionosphere (at least in the earthward quadrant of the Hall region). These electrons are needed for current closure and for maintaining the quasineutrality condition. However, this boundary is probably less sharp because the inertia of the lobe ions which have moved into the current sheet is still large enough for continuing their motion over a substantial part of their path even though they have become unmagnetized unless some mechanism acts which causes thermalization and dissipates their convective momentum. In the purely collisionless plasma there is no obvious mechanism which could cause such a retardation of the ions. Instead it is the continuous increase in the electron convection velocity $E_z/B_y$ during their convective displacement into the weak magnetic field region around the X-line which separates the electrons from the ions. In addition the lower hybrid drift waves in the boundary layer of the current sheet scatter the ions perpendicular to the magnetic field on a time scale of several lower hybrid periods. In the tail magnetic field of a few nT, the lower hybrid period is of the order of $\lesssim 1$ s. Thus, it takes several seconds for the ions to lose their convective momentum in this process. Hence, on the lobe side of the ion inertial range the ions still traverse a substantial fraction of the ion inertial length until their motion separates from that of the electrons, and the region of upward flowing electrons, respectively, downward field-aligned currents shrinks in comparison with the region of downward electrons, respectively, upward field-aligned currents.

The above expression for the field-aligned current supports the view of the direction of the current. Sufficiently far away from the electron-diffusion region $n/B \approx \text{const}$, and the direction of the parallel current is determined by the sign of $V(1/B)$. Since $B$ decreases from the lobes into the ion-diffusion region on the inflow side and increases out of the ion-diffusion region on the outflow side, one concludes that in the inflow and outflow regions the directions

---

Fig. 6. Left: Current and electric field system in a substorm related upward–downward current system $\pm j_{||}$. As in the observations of Fig. 5 the upward current is flanked by two regions of downward currents. The red lines are equipotentials showing the development of upward and downward localized electric fields $\pm E_{||}$ in the auroral region. Right: Schematics of a small scale tailward reconnection site that is magnetically connected to the auroral region. In this picture it is assumed that the magnetic connection between the narrow current filament and the aurora zone is straight and no topological problems arise. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)
of the field-aligned currents are opposite. This is in agreement with the requirement that the field-aligned currents close the divergent Hall current system. The auroral upward and downward current regions are connected to the image of the ion inertial region in the tail reconnection sites as shown schematically in Fig. 6. These are very small, the order of roughly $100–1000$ km extension. Mapped down to the ionosphere the whole system of upward–downward currents appears of rather short extension in north–south direction, only a few $10$ km wide. This can be the case only if very thin current layers or filaments in the tail are involved. Reconnection taking place in the tail should occur in many such small reconnection events distributed over a substantial fraction of the near-Earth volume of the plasma sheet during substorms. This is suggested by the smallness of both the reconnection diffusion and the auroral acceleration regions (Carlson et al., 1998) as observed by FAST (for a recent analysis of FAST data and their fine structure see, e.g., McFadden et al., 2003; Pottelette et al., 2004) and by the abundance of crossings of inverted-Vs in the auroral upper ionosphere. Such a view would be consistent with suggestions (see, e.g., Tetreault, 1990; Chang et al., 2004) that reconnection in the magnetosphere during substorms is to a certain extent of statistical nature consisting of many small reconnection sites that are connected to the local current systems in the auroral ionosphere. In spite of its attractiveness this model encounters severe topological difficulties resulting from the requirement that magnetic field lines must close. It can be realized only in manifestly three-dimensional reconnection in Earth’s magnetotail. When such a model is valid the plasma sheet in the tail during substorms will be filled with a large number of current filaments, small magnetic island and X-points, some of them connected to the ionosphere in order to provide the upward and downward field-aligned current system.

In concluding we note that a large number of important and unresolved questions remain concerning the role of the Hall effect in reconnection. These concern the reconnection topology, the relation to the auroral current system, the question of how the ionospheric reservoir of cold electrons is informed about the need to close the Hall currents in the reconnection region and others more. The answer to the latter question is probably related to the motion of the magnetized electrons across the immobile ions on the scale of $\lambda_i$ or a fraction of it which is the transverse scale of kinetic Alfvén waves. Kinetic Alfvén waves should be involved into the transport of information from the Hall-diffusion site into the auroral ionosphere. This transport happens within the allotted convection time across the Hall region and thus limits the efficiency of generation of field-aligned currents. At present these processes are barely understood.

Another problem is the still badly understood breaking of the frozen-in condition. The Hall effect shows that the ions can indeed become unmagnetized. For the electrons no simple way exists to demagnetize them on the same scale. One has to refer to other badly understood effects like non-diagonal pressure anisotropies, electron inertial effects, and instabilities which scatter the electrons away from the magnetic field—or to diamagnetic effects (Treumann et al., 2004) which naturally destroy magnetic fields locally in similarity to “superconductivity” in condensed matter physics. Some of these questions will be considered elsewhere.

Acknowledgements

The Cospar-Colloquium in Israel with its stimulating scientific climate was a very fruitful and enjoyable event. We thank M. Balikhin, W. Baumjohann, D. Eichler, R. Ergun, M. Gedalin, E. Georgescu, S. Haaland, R. Pottelette, B. Sonnerup, A. Vörös, and M. Volwerk for discussions. Part of this work benefited from a visiting scientist period of RT at ISSI. The hospitality of its directors, R. Bonnet, J. Geiss, G. Paschmann, and R. von Steiger, as well as the hospitality of ISSI and its staff are gratefully acknowledged. An unknown referee provided valuable remarks on the original version of this paper.

References


Parker, E.N. Sweet’s mechanism for merging magnetic fields in conducting fluids. J. Geophys. Res. 62 (11), 509–520, 1957.


