MHD model of the flapping motions in the magnetotail current sheet

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A new kind of magnetohydrodynamic waves is analyzed for a current sheet in the presence of a small normal magnetic field component \( (B_z) \) varying along the sheet. For the initial undisturbed state, a simplified model of the current sheet is considered with a Harris-like current density distribution across the sheet. Within the framework of this model, an analytical solution is obtained for the flapping-type wave oscillations and instability, related to the gradient of the normal magnetic field component along the current sheet. The flapping wave frequency is found to be a function of the wave number, which has an asymptotic saturation for large wave numbers. This frequency is pure real in a stable situation for the magnetotail current sheet, when the \( B_z \) component increases toward Earth. The current sheet becomes unstable in some regions, where the \( B_z \) component decreases locally toward Earth. In the stable region, the “kink”-like wave oscillations are calculated for an initial Gaussian perturbation localized to the center of the current sheet. The flapping wave propagations are analyzed for two cases: (1) the initial perturbation is fixed, and (2) the source is moving toward Earth. In the last case, the Mach cone is obtained for the propagating flapping waves. The source for the flapping waves is associated with the fast plasma flow originated from the reconnection region.


1. Introduction

[1] Flapping oscillations of the magnetotail current sheet are very interesting phenomena indicated by many spacecraft measurements. In particular, CLUSTER observations in the Earth’s magnetotail current sheet showed the appearance of strong wave perturbations propagating along the current sheet perpendicular to the magnetic field lines. Observed cases of such wave perturbations were first discussed by Zhang et al. [2002]. Statistical studies of Sergeev et al. [2003, 2004], Runov et al. [2005a, 2005b, 2006], and Petrukovich et al. [2006] confirmed the existence of such kind of waves identified as “kink”-like perturbations. The CLUSTER observations give rise to the assumption that the flapping motions are notably more frequent in the central part of the tail than near the flanks. This is also confirmed by Geotail observations [Sergeev et al., 2006]. In the near-flank tail regions, a propagation of the flapping waves occurs predominantly from the center to the flanks [Sergeev et al., 2004]. These experimental results suggest an internal origin of the flapping motions, due to some processes (like magnetic reconnection) localized deep inside the magnetotail. The plasma sheet flapping observations are interpreted as crossings of a quasi-periodic dynamical structure produced by an almost vertical slippage motion of the neighboring magnetic flux tubes [Petrukovich et al., 2006]. The frequency of the flapping motions estimated from observations is about \( \omega_f \sim 0.035 \text{ s}^{-1} \) [Sergeev et al., 2003]. For the majority of the observed events [Runov et al., 2005a], the group speed of the flapping waves is within the range of a few tens (30–70) kilometers per second. The wavelengths and spatial amplitudes are of the order of 2–5 \( R_E \) (\( R_E \) is the Earth’s radius) [Petrukovich et al., 2006]. On the basis of CLUSTER observations of reconnection events, a relationship between the flapping motions and the reconnection process was investigated by Laitinen et al. [2007]. During reconnection events, the current sheet exhibits strong flapping wave oscillations that propagate toward the flanks of the magnetotail.

[2] In spite of a substantial advance in statistical studies of the flapping oscillations, the physical mechanism of this effect has not been understood well. Global magnetohydrodynamic (MHD) simulations can reproduce usually only large scale features, but they do not have sufficient resolution to describe such small scaled structures like flapping wave disturbances in the magnetotail. Therefore, specific
physical models are required for interpretation of these events.

[4] There exist several theoretical models for flapping waves in a current sheet. In particular, the Ballooning-type mode in a curved current sheet magnetic field was claimed to be able to propagate azimuthally in flankward directions from the source [Golovchanskaya and Maltsev, 2005]. This ballooning theory was applied in the WKB approximation implying the condition that the wave length scale is much less than the curvature radius of the magnetic field line. But this condition can be fulfilled only in the near Earth edge of the plasma sheet. Another point is that, according to the theory of Golovchanskaya and Maltsev [2005], both “kink”-like and “sausage”-like deformations of the current sheet are equally possible, and the question arises about the reason, why the observed flapping perturbations of the current sheet are mainly associated with the “kink”-like wave modes.

[5] Volwerk et al. [2003] tried to interpret flapping oscillations of the current sheet, observed before and after substorm onset, as driven magnetoacoustic “kink” modes which were studied by Smith et al. [1997]. However, magnetoacoustic waves have too large frequency and wave speed, exceeding the observed values.

[6] Theoretical investigation of Wang et al. [1988] described the “kink”-like oscillations propagating along the current sheet in the direction perpendicular to the electric current, which are different from those of flapping modes considered in our study.

[7] A drift “kink” mode due to a relative drift of electrons and protons was proposed by Daughton [1999] to interpret the flapping oscillations. But for high mass ratio this mode has a very weak growth. Another possibility is to apply the ion/lon drift “kink” mode studied by Karimabadi et al. [2003a, 2003b] and Sitnov et al. [2004], which has larger growth rate depending on the ion bulk-flow velocity shear. This mechanism seems to be rather effective for very thin current sheets.

[8] Recently, a new approach within the framework of MHD modeling was proposed by Erkaev et al. [2007, 2008]. It was found that the MHD flapping modes can appear due to the gradient of the normal magnetic field component along the current sheet.

[9] In this paper, we develop further the model of Erkaev et al. [2007, 2008]. In particular, we apply a more realistic Harris-like profile for the initially undisturbed current density, and also we analyze the wave propagations from fixed and moving sources at the center of the current sheet.

2. Basic Equations and Model Assumptions

[10] We apply the system of incompressible ideal magnetohydrodynamics (MHD) for nonstationary variations of the plasma sheet parameters

\[ \rho \left( \frac{\partial \mathbf{V}}{\partial t} - \mathbf{V} \cdot \nabla \mathbf{V} \right) + \nabla P = \frac{1}{4\pi} \mathbf{B} \cdot \nabla \mathbf{B}, \]  

\[ \frac{\partial \mathbf{B}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{V}, \]  

\[ \nabla \cdot \mathbf{V} = 0, \quad \nabla \cdot \mathbf{B} = 0. \] 

Here, \( \mathbf{V}, \mathbf{B}, \rho, P \) are the velocity, magnetic field, density and total pressure, respectively. The total pressure is defined as the sum of the magnetic and plasma pressures. We consider specific wave perturbations propagating across the magnetic field lines, which are much slower than the magnetosonic modes. In this case, the incompressible approximation seems to be appropriate. We focus our study on very slow wave modes existing only in the presence of a gradient of the \( B_z \) component in the magnetotail current sheet along the \( x \) direction. A magnetic field configuration and coordinate system are shown in Figure 1. Here, the \( B_z \) component is assumed to have a gradient along the \( z \) direction, and thus we consider the two magnetic gradients as key factors for the current sheet oscillations. This approach, applied by Erkaev et al. [2008] for the flapping wave oscillations, was called the “Magnetic double gradient mechanism.”

[11] We begin with a qualitative explanation of the expected MHD flapping instability and waves. Let us consider a plasma element of a unit volume at the center of the current layer as shown in Figure 1. The resulting force \( F_z \), acting on this plasma element along the \( z \) direction, is the difference of two forces caused by the magnetic tension and the total pressure gradient. In equilibrium state, the resulting force \( F_z \) vanishes, and the total pressure gradient compensates the magnetic tension,

\[ \frac{\partial P}{\partial z} = \frac{1}{4\pi} B_z \frac{\partial B_z}{\partial x}. \]  

A small displacement of the magnetic tube element along the \( z \) axis yields the restoring force

\[ F_z = -\frac{1}{4\pi} B_z(\hat{z} \delta z) \frac{\partial B_z}{\partial x} = -\frac{1}{4\pi} \hat{z} \delta z \left( \frac{\partial B_z}{\partial z} \frac{\partial B_z}{\partial x} \right)_{z=b}. \] 

Here, \( B_z(\hat{z} \delta z) \) is determined from a Taylor series expansion. This force accelerates the plasma in the \( z \) direction,

\[ \frac{\partial^2 \delta z}{\partial t^2} = -\omega_b^2 \delta z, \] 

where \( \omega_b \) is the buoyancy frequency.
Parameter $\omega_f$ is real in case of a positive product of the two magnetic gradients, and it has the meaning of a characteristic frequency of the flapping wave oscillations. In case of a negative product of the magnetic gradients, $\omega = i\gamma$, and thus the current sheet is unstable. This situation describes an exponential growth of the flapping perturbation without propagation. The initial undisturbed total pressure profiles are very different for stable and unstable current sheets. Namely, at the center of the current sheet, the total pressure has a maximum for the unstable situation, and it has a minimum for the stable conditions. This means that the flapping effect can not be described within the framework of the boundary layer approximation, assuming the constancy of the total pressure across the current sheet [Schindler, 2007]. The variation of the initial undisturbed total pressure along the normal direction with respect to the current sheet is a very important factor for the flapping oscillations.

[12] A qualitative explanation of the wave oscillations and the instability is illustrated in Figure 2, where Figures 2a and 2b correspond to stable and unstable situations, respectively. Figure 2b illustrates the case of a local thinning of the current sheet.

[13] Furthermore, we take into consideration the MHD equations (1)–(3) and find solutions corresponding to the qualitative scheme as discussed above. The initial current sheet configuration, shown in Figure 3, is considered to be rather simple with a weak dependence of the $B_z$ component on the $x$ coordinate

$$B = [B_z(z/\Delta), 0, B_z(x/L_x)], \quad V = 0.$$  

(8)

Here $\Delta$ is the half-thickness of the current sheet, and $L_x$ is the length scale of the $B_z$ variation along the current sheet. We introduce two dimensionless parameters, $\epsilon = B_z(0)/B_{z\text{ max}}$ and $\nu = \Delta/L_x$, which are assumed to be small.

[14] We consider small perturbations of the magnetic field, total pressure, and velocity,

$$B = (B_x + b_x, b_y, B_z + b_z), \quad P = P_0 + p, \quad V = (v_x, v_y, v_z).$$  

(9)

[15] Linearizing equations (1)–(3) for small perturbations, we neglect small terms $B_z \nabla_z b_z$ and $B_z \nabla_z b_y$ ($\nabla_z$ is the partial derivative with respect to the axis $z$), and retain the main term $b_z \nabla_z B_z$. This is justified under the condition $B_z L_x/B_y(\Delta) = \epsilon/\nu \ll 1$. This condition seems to be appropriate for a steady-state magnetotail current sheet. In particular, it is valid for the analytical current sheet equilibrium solutions of Kan [1973] and Manankova et al. [2000] presented in Figures 4 and 5, respectively. In Figure 4, related to Kan [1973], are shown from top to bottom the magnetic field lines, the $B_z$ component as a function of $z$, the $B_z$ component and the ratio $\epsilon/\nu$ as functions of $x$, and the total pressure profile. The $B_z$ and $P$ profiles are given for $x = 7$ indicated by asterisks in Figure 4 (top). Figure 5, related to Manankova et al. [2000] shows the magnetic field lines, the $B_z$ component as a function of $z$, the $B_z$ component as a function of $x$, and the total pressure profiles. The profiles along the $z$ axis are given for three $x$ values: $x_1 = 7$ (red), $x_0 = 9$ (blue), $x_1 = 12$ (green), marked by the corresponding asterisks in Figure 5 (top). All quantities in Figures 4 and 5 are given in dimensionless units.

[16] We also use the simplifying assumption that all wave perturbations propagating in the $y$ direction do not depend on the $x$ coordinate, and thus they are considered to be functions of time and two Cartesian coordinates $(y, z)$. The linearized equations with the underlined terms to be neglected are the following

$$\rho \frac{\partial v_y}{\partial t} + \frac{\partial P}{\partial z} = \frac{1}{4\pi} \left( \frac{\partial B_z}{\partial x} + b_x \frac{\partial B_z}{\partial z} + B_z \frac{\partial B_z}{\partial x} + b_z \frac{\partial B_z}{\partial z} \right),$$

$$\rho \frac{\partial v_z}{\partial t} + \frac{\partial P}{\partial z} = \frac{1}{4\pi} \left( \frac{\partial B_z}{\partial x} + b_x \frac{\partial B_z}{\partial z} + B_z \frac{\partial B_z}{\partial x} + b_z \frac{\partial B_z}{\partial z} \right),$$

$$\frac{\partial v_z}{\partial z} - v_z \frac{\partial v_z}{\partial x} = 0,$$

$$\frac{\partial b_z}{\partial t} = -B_z \frac{\partial v_z}{\partial x} - B_z \frac{\partial v_z}{\partial x}.$$  

(10)

[17] In order to check the relevance of the model assumption as addressed above for a real situation, we used the

Figure 3. Geometrical situation of the problem: The initial undisturbed current sheet configuration.
Figure 4. Solution of Kan [1973]: Magnetic field lines, the $B_x$ component as a function of $z$ for $x = 7$, the $B_z$ component and ratio $\epsilon/\nu$ as functions of $x$, and the total pressure profile for $x = 7$. All quantities are given in dimensionless units.

Figure 5. Solution of Manankova et al. [2000]: Magnetic field lines, the $B_x$ component for $x = 7$ (red), 9 (blue), 12 (green), the $B_z$ component as a function of $x$, and the total pressure profiles. All quantities are given in dimensionless units.
model of Tsyganenko (T96) with parameters $P_{dsw} = 2$, Dst = 10, $B_{ysw} = 2$, $B_{zsw} = 2$ to determine the ratio $\varepsilon/\nu = (B_z/B_x)(L_x/D)$ as a function of the $x$ coordinate along the magnetotail. The variation of this ratio is shown in Figure 6 and indicates that the condition $\varepsilon/\nu /C_{28} < 1$ is valid in the region $25 \leq x \leq 10 \times 10^7$, where the model T96 is applicable.

3. Linear Analysis of Eigenmodes

[18] Assuming Fourier harmonics ($\exp(i\omega t - iky)$) in equation (10) and neglecting the underlined terms, we obtain finally the system of equations for the Fourier amplitudes

$$i\omega \nu v_x = \frac{1}{4\pi} \left( b_x \frac{dB_y}{dz} + b_y \frac{dB_x}{dz} \right),$$

$$i\omega \nu v_y - ikp = 0, \quad i\omega \nu v_z + \frac{dp}{dz} = \frac{1}{4\pi} b_y \frac{dB_z}{dx},$$

$$i\omega b_z - B_z \frac{dv_z}{dz} + v_x \frac{dB_z}{dx} = 0, \quad i\omega b_y - B_y \frac{dv_y}{dz} = 0,$$

$$i\omega b_x - \frac{dB_x}{dz} v_z = 0, \quad -ikv_x + \frac{dv_x}{dz} = 0.$$

[19] In this system of equations, the derivative $dB_z/dx$ is assumed to be constant, and all other quantities are considered to be not dependent on the $x$ coordinate. From these linearized equations, treated as a system of ordinary equations with respect to $z$, we finally obtain a second order ordinary differential equation for the $v_z$ velocity perturbation

$$\frac{d^2 v_z}{dz^2} + k^2 v_z \left( U(z) - 1 \right) = 0,$$  \hspace{1cm} (15)

where

$$U(z) = \frac{1}{4\pi \rho} \frac{\partial B_y}{\partial z} \frac{\partial B_z}{\partial x}.$$  \hspace{1cm} (16)

[20] For a Harris-like variation of the undisturbed magnetic field

$$B_e = B^* \tanh(z/\Delta),$$  \hspace{1cm} (17)

function $U(z)$ has a hyperbolic behavior

$$U(z) = \frac{B^*}{4\pi \rho \Delta} \frac{\partial B_z}{\partial x} \frac{1}{\cosh(z/\Delta)^2}. $$  \hspace{1cm} (18)

[21] Equation (15) is similar to that known from the theory of tearing mode instabilities [Pritchett et al., 1991]. With function (18), the spectral problem for equation (15)
has analytical solutions corresponding to “kink”-like and “sausage”-like modes. The eigenfunctions are expressed via the Legendre functions \(P_l^m\) as follows

\[
v_z = C P_l^m \frac{\tanh z}{D} = \frac{l}{C_0} \frac{1}{2 + \frac{1}{4} + kD w_f}.
\]

where

\[
\omega_f = \sqrt{1 + \frac{B_x^2}{4\pi\rho} \frac{\partial B_z}{\partial x} \frac{\partial B_z}{\partial x}}.
\]

For the “kink” mode, \(v_z\) is an even function of the \(z\) coordinate, which requires to fulfill condition \(\lambda = -\mu = k\Delta\). This relation between \(\lambda\) and \(k\) yields the equation

\[
k\Delta = -1/2 + \sqrt{1/4 + (k\Delta)^2 \omega_f^2/\omega^2}.
\]

From this equation we derive frequency and group speed as functions of the wave number for the “kink” mode

\[
\omega_k = \omega_f \sqrt{\frac{k\Delta}{k\Delta + 1}},
\]

\[
V_{gk} = \omega_f \Delta F(k\Delta),
\]

\[
F(k\Delta) = \frac{1}{2\sqrt{k\Delta(1 + k\Delta)}}.
\]

[22] For the “sausage” mode, \(v_z\) is an odd function, which vanishes at the center of the current sheet. This mode corresponds to the condition \(\lambda = k\Delta + 1\) which leads to

\[
k\Delta + 3/2 = \sqrt{1/4 + (k\Delta)^2 \omega_f^2/\omega^2}.
\]

This equation determines the “sausage” mode frequency and group speed as explicit functions of the wave number

\[
\omega_s = \omega_f \frac{k\Delta}{\sqrt{(k\Delta)^2 + 3(k\Delta) + 2}},
\]

\[
V_{gs} = \omega_f \Delta \frac{3k\Delta + 4}{2\sqrt{(k\Delta)^2 + 3(k\Delta) + 2}}.
\]

[23] The normalized frequencies \(\omega_k/\omega_f\) and wave group velocities are presented in Figure 7. Figure 7 shows that the flapping wave frequency is maximal for short wave length perturbations which propagate much slower than those with long wave lengths.

**Figure 7.** Frequencies and group velocities as functions of the wave number for the “kink” and “sausage” wave modes.
The “kink” perturbations grow faster than the “sausage” ones. This unstable situation can be realized at some parts of the Earth’s magnetotail current sheet, where the $B_z$ component has a local decrease toward Earth.

[26] For example, we estimate the flapping frequency for the parameters that seem to be reasonable for the current sheet conditions in the Earth’s magnetotail,

$$B_x = 20 \text{nT}, \ B_z = 2 \text{nT}, \ \Delta \sim R_E, \ n_p = 0.1 \text{ cm}^{-3},$$

$$k\Delta = 0.7, \ \partial B_z/\partial x \sim B_z/L_x, \ L_x \sim 5R_E.$$

(30)

For these parameters we find the characteristic flapping frequency $\omega_f \sim 0.03 \text{ s}^{-1}$, and also the group velocity $V_g = 60 \text{ km/s}$.

4. Flapping Wave Perturbations Induced by a Source

[27] We consider an initial “kink”-like Gaussian perturbation $\xi$ of the current sheet which is produced by some external source

$$\xi(y,0) = \xi_0 \exp(-\sigma y^2),$$

(31)

Figure 8. Eigenfunctions corresponding to the “kink” mode: Profiles of the normalized velocity components ($v_z, v_y$) and perturbations of the magnetic field components ($b_z, b_y$).

Figure 9. Eigenfunctions corresponding to the “sausage” mode: Profiles of the normalized velocity components ($v_z, v_y$), and perturbations of the magnetic field components ($b_z, b_y$).

\[ \tau_k = \tau_f \sqrt{1 + 1/(k\Delta)}, \]

(28)

\[ \tau_s = \tau_f \sqrt{1 + 3/(k\Delta) + 2/(k\Delta)^2}, \]

(29)

\[ \tau_f = \left( \frac{-1}{4\pi \rho} \frac{\partial B_z}{\partial x} \frac{\partial B_z}{\partial \rho} \right)^{-1/2}. \]
where $x_0$ and $s$ are the parameters characterizing the amplitude and length scale of the initial disturbance, respectively.

Using the Fourier method and the dispersion equation (22), we find the solution for the initial condition (31)

$$
\xi(y, t) = \int_{-\infty}^{\infty} A(k) \exp[i(\omega(k)t - ky)]dk,
$$

where function $A(k)$ is given by the Fourier integral

$$
A(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \xi(y, 0) \exp[iky]dy.
$$

This solution is presented in Figure 10 (top and middle) which shows perturbations of the magnetic surfaces in the current sheet in the cases of “kink”-like and “sausage”-like flapping oscillations. These wave oscillations, produced by the initial perturbation (31), propagate toward the flanks of the current sheet. At the center of the current sheet, there are many oscillation pulses produced by one initial pulse. The damping of these oscillations is rather slow.

Next we modify the statement of problem considered in the previous section for a moving source, which can be a fast magnetic flux tube originating from a magnetic reconnection region. The flapping wave disturbances, produced by a moving source, has to be bounded by a cone with angle $\theta$

$$
\theta = \arctan\left(1/M_f F(k\Delta)\right),
$$

where $M_f$ is the flapping Mach number determined as the ratio of the source velocity ($V_s$) to the characteristic wave propagation speed given by formula (23),

$$
M_f = \frac{V_s}{\omega_f \Delta}.
$$

This is an important dimensionless similarity parameter for the flapping waves (like Alfvén and sonic Mach numbers for magnetosonic waves), which characterizes the interaction between a moving source and flapping waves in the current sheet.

Figure 10 (bottom) shows wave disturbances of the current sheet, which are initiated by a source rapidly moving toward Earth (hypothetically “bursty bulk flow” or BBF). Figure 10 indicates the Mach cone, which determines the propagation of the flapping wave oscillations. Figure 10 (bottom) corresponds to the flapping Mach number equal to 1. Figure 10 (top and middle) shows the perturbation of the magnetic surface corresponding to “kink”-like and “sausage”-like modes induced by a fixed initial Gaussian pulse at the center of the current sheet.

Next we estimate the energy loss of a BBF due to the excitation of flapping waves. The kinetic energy of the BBF is estimated as

$$
W_{bbf} = \frac{1}{2} \rho \ell_x \ell_y \ell_z V^*^2,
$$

where $\ell_x$, $\ell_y$, $\ell_z$ are the sizes of the BBF in $x$, $y$, $z$ directions, and $V^*$ is the velocity of the BBF. The decrease of the BBF kinetic energy is equal to the energy flux of the flapping wave. This energy balance yields the equation

$$
\frac{dW_{bbf}}{dx} V^* \ell_y \ell_z = 2W_f V_g \ell_x \ell_z,
$$

where $W_f$ is the energy density of the flapping waves propagating into the two directions toward the flanks, $W_f \sim \rho V^2/2$ ($v$ is the wave amplitude), and $V_g$ is the group speed of the flapping wave. From this equation, we estimate a characteristic length of deceleration of the BBF,

$$
L^* = \frac{V^*^2}{V_f V_g} = \frac{\ell_y \ell_z}{2 W_f V_g} V^*^2 / v^2.
$$

Taking reasonable parameters, $V^* \sim 400$ km/s, $v \sim 60$ km/s, $V_g \sim 60$ km/s, $\ell_y \sim 2 R_E$, we find $L^* \sim 300$ $R_E$. This deceleration length is much larger than the distance covered by the BBF during its travel to the Earthward edge of the current sheet. This yields to the conclusion that the BBF
spends a small part of its energy for the excitation of flapping oscillations. The estimated flapping Mach number is about $M_f = V^*/V_g \sim 7$. This means that the real cone angle during BBF propagation is much smaller than that shown in Figure 10. In Figure 10 we applied a smaller flapping Mach number and, correspondingly, the larger cone angle, for which the flapping wave oscillations are more visible.

[32] Finally we estimate the necessary condition for the applicability of an incompressible plasma model. The total pressure perturbation can be estimated as $\delta P = \rho \omega^2 \kappa$. The perturbation of the plasma pressure is considered to be of the same order as that of the total pressure, $\delta p = \rho \omega^2 \kappa$. The plasma density and pressure perturbations are proportional to each other, $\delta \rho = \delta p/c_s^2 < \rho \omega^2 (c_s^2 k)$, where $c_s$ is the sonic speed. Then the time derivative of the plasma density can be estimated as

$$\left(\frac{\partial \delta \rho}{\partial t}\right) \sim \left(\omega/kc_s\right)^2 (\rho, k).$$

(38)

This density derivative term can be neglected in the continuity equation in the case of condition $\omega/kc_s \ll 1$, which means that the phase speed of wave perturbations is much less than the sonic speed. For the flapping waves this condition is well satisfied.

5. Summary

[33] Within the conventional MHD approach, flapping waves and instability are analyzed in application to the magnetotail current sheet in the presence of a normal magnetic field component ($B_z$), which has a weak variation along the $x$ direction. The main difference from previous work [Erkaev et al., 2008] is that we consider a Harris-like current density profile, which yields a quite simple analytical expressions for the flapping frequency as a function of the wave number, corresponding to the “kink” and “sausage” modes. This profile seems to be more realistic than the piecewise constant and parabolic profiles considered in the previous work. For the Harris-like profile we found smaller flapping frequency than for the others.

[34] We present also the eigenfunctions describing the velocity and magnetic field perturbations as functions of the $z$ coordinate. For the “kink” mode, the maximum of the $B_y$ perturbation coincides with the $v_z$ maximum at the center of the current sheet. The ratio of the $B_y$ and $v_z$ maximal amplitudes is about 1.4 $B^*/(\omega D)$. For this mode, the $B_y$ and $v_z$ perturbations vanish at the center of the current sheet, and they reach maximal values at the edge of the sheet.

[35] The obtained analytical dispersion function was used to calculate the propagation of flapping waves induced by given sources localized to the center of the current sheet.

[36] For typical parameters of the Earth’s current sheet, the group velocity of the “kink”-like mode is estimated to be a few tens of kilometers per second, that is in good agreement with CLUSTER observations. A strong decrease of the group velocity for high wave numbers means that the small scale oscillations propagate much slower than the large scale ones. Because of that, the propagating flapping pulse has a shallow front side part, and a small scale oscillating backside part.

[37] Magnetic gradients $\nabla B_z$ and $\nabla_x B_z$ play a crucial role for the stability of the current sheet. It can be stable or unstable, if the product of these two magnetic gradients is positive or negative. In particular, the instability can arise in the vicinity of a localized thinning of the current sheet (Figure 2), where the $B_z$ component decreases toward Earth (has an inverse gradient). In this unstable situation, the frequency is pure imaginary, and the flapping perturbations just grow up without propagation. These growing-up “kink”-like disturbances of the current sheet can be embedded into the plasma bulk flow. A nonlinear saturation for the unstable modes is expected, if the wave displacement becomes of the order of the distance between the local minimum and maximum of the total pressure. In accordance to Figure 4, this distance is about the current sheet thickness.

[38] A stable situation takes place for the current sheet with the $B_z$ component increasing toward Earth. In this case, flapping waves can be initiated by a source related to a magnetic reconnection process in the magnetotail. This source can bring the initial “kink”-like disturbances out from the reconnection region, where the flapping instability takes place. A quite reasonable physical scenario could be the following. A magnetic reconnection event produces a reconnected magnetic flux tube, which moves very rapidly through the center of the current sheet toward Earth. This magnetic tube induces flapping wave disturbances of the current sheet like a ship, moving on the water surface. The existence of such accelerated magnetic tubes (BBFs) is well known [Angelopoulos et al., 1992]. Such a scenario is consistent with the finding that flapping events and BBFs have similar radial and azimuthal frequency distributions in the magnetotail [Sergeev et al., 2006].

Assuming this scenario, we analyzed a propagation of flapping waves induced by a model Gaussian source localized to the center of the current sheet. For these flapping waves, we obtained a frequency and a spatial distribution of the wave amplitude. The flapping wave disturbances, produced by the moving source, are found to be bounded by the Mach cone depending on the special flapping Mach number defined as the ratio of the source velocity to the characteristic wave propagation speed.

[39] In our model, a guide field is not taken into account. With a guide field, the wave frequency and group speed are expected to be larger. And also, a guide field will lead to a threshold for the flapping instability.

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