Dissipation of Alfvén wave pulses propagating along dipole magnetic tubes with reflections at the ionosphere

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Abstract

A ratio of the maximal and minimal cross sections of the magnetic tube (contraction ratio) is a crucial parameter which affects very strongly on reflections of MHD wave pulses propagating along a narrowing magnetic flux tube. In cases of large contraction ratios of magnetospheric magnetic tubes, the wave energy flux at the ionospheric boundary can be rather small. Therefore the dissipation of the wave perturbations can be very weak for each reflection, in spite of a finite conductivity of the planet's ionosphere. The dissipation is stronger for the pulses with shorter wave scales. Because of that, Alfvén wave pulses with sufficiently long wave scales have a very small energy loss for each reflection at the conducting ionosphere, and thus, they have many reflections without a noticeable decrease of their amplitude. This effect related to converging magnetic lines is dependent very strongly on the polarization of the Alfvén wave. In case of a dipole magnetic field, the effect is most pronounced for wave pulses characterized by velocity and magnetic perturbations in the meridional plane.

Keywords: Alfvén wave pulses; Converging magnetic field lines; Reflections at the ionosphere

1. Introduction

In the general case, the problem of propagation of magnetohydrodynamic waves in a nonuniform magnetic field is complicated to solve. To simplify the problem, a fruitful assumption of large azimuthal wave numbers was proposed in a series of publications (Southwood and Saunders, 1985; Walker, 1987; Leonovich and Mazur, 1993; Klimushkin, 1998) which implies the longitudinal wave length to be much larger than the azimuthal one. In this approach, the fast magnetosonic mode is strongly evanescent, and only transverse Alfvén waves coupled to slow mode magnetosonic waves can be described. In such a case the perturbation of the total pressure is zero, and thus this assumption is relevant to a thin magnetic tube approximation which considers the normalized thickness of the magnetic flux tube to be a small parameter (Rae and Roberts, 1982). The condition of such an approach is that the total pressure (the sum of magnetic and plasma pressures) is constant across the magnetic tube. These models are applicable for magnetic flux loops in the solar corona and also for disturbed magnetic tubes in the magnetospheres of Earth and other planets. In particular, the model of a thin magnetic tube was applied for slow magnetosonic pulses propagating along a dipole magnetic tube in the Jovian magnetosphere (Erkaev et al., 2002).

Using the approach of large azimuthal wave numbers, we analyze the propagation of Alfvén wave pulses along a nonuniform magnetic tube, and in particular we focus attention on the effects related to the converging of magnetic field lines. The main aspect of our study is the influence
of the magnetic field converging on the dissipation of Alfvén pulses reflecting from the ionospheric boundary with a finite conductivity.

2. Basic equations

In the dissipationless approximation, the magnetic field and plasma parameters are determined by the ideal MHD equations which are commonly used for space plasmas,

$$\frac{\partial \mathbf{V}}{\partial t} + \rho (\mathbf{V} \cdot \nabla) \mathbf{V} + \nabla \Pi - \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B} = 0,$$

$$\Pi = P + B^2/(8\pi), \quad \frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{V}) = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} - \text{rot} (\mathbf{V} \times \mathbf{B}) = 0, \quad \text{div} \mathbf{B} = 0.$$

Here $\rho$, $\mathbf{V}$, $P$, $\mathbf{B}$ are mass density, velocity, plasma pressure and magnetic field, respectively, $\Pi$ is a total pressure (the sum of the magnetic and plasma pressures), and $\gamma$ is the polytropic index.

The geometrical situation of the statement of the problem is illustrated in Fig. 1. Here $\rho_0$ is the radius of the ionosphere, and $r_0$ is the distance to the equatorial point of the magnetic tube, where the pulse is initiated. The initial perturbation is given in Gaussian form at the equator.

Distance $(r)$, time $(t)$, plasma parameters and magnetic field are normalized as follows:

$$\tilde{r} = \frac{r}{r_0}, \quad \tilde{t} = \frac{t}{r_0}, \quad \tilde{\rho} = \frac{\rho}{\rho_0}, \quad \tilde{P} = \frac{4\pi}{B_0^2},$$

$$\tilde{\mathbf{B}} = \frac{\mathbf{B}}{B_0}, \quad \tilde{\mathbf{V}} = \frac{\mathbf{V}}{V_0},$$

where $V_0$ is the Alfvén speed, $\rho_0$ and $B_0$ are the background mass density and magnetic field strength at the equatorial point of the magnetic tube.

Hereafter, the magnetic field is assumed to be axisymmetric. Perturbations of a magnetic flux tube can be described in terms of suitable orthogonal curvilinear coordinates $\sigma$, $\lambda$, $\zeta$ related to a given magnetic tube. We consider the first variable to characterize the displacement of a fluid particle along the magnetic field line. The second and the third variables correspond to independent displacements which are perpendicular to the magnetic field. The total squared displacement is determined by the quadratic form

$$d^2 = g_1 d\sigma^2 + g_2 d\lambda^2 + g_3 d\zeta^2,$$

where $g_1$, $g_2$ and $g_3$ are the metric coefficients which are expressed through the position vector derivatives with respect to the variables $(\lambda, \sigma, \zeta)$. We introduce the material coordinates which are based on the equations $\mathbf{B} \cdot \nabla \phi = 0$, $\mathbf{B} \cdot \nabla \psi = 0$, $\mathbf{B} \cdot \nabla \zeta = \rho$. Quantities $\phi$ and $\psi$ can be chosen as Euler potentials for the magnetic field, and quantity $\zeta$ is a function of the distance $\sigma$ along the tube which identifies the distribution of the mass of the plasma in the magnetic tube with unit flux.

The magnetic field strength and velocity squared are determined by the derivatives of the total displacement with respect to $t$ and $x$. Using Eq. (6) we obtain

$$\tilde{\mathbf{V}}^2 = \tilde{g}_1 \tilde{\sigma}^2 + \tilde{g}_2 \tilde{\lambda}^2 + \tilde{g}_3 \tilde{\zeta}^2,$$

$$\tilde{\mathbf{B}}^2 = \tilde{\rho}^2 (\tilde{g}_1 \tilde{\sigma}^2 + \tilde{g}_2 \tilde{\lambda}^2 + \tilde{g}_3 \tilde{\zeta}^2).$$

Here subscripts “$t$” and “$x$” denote derivatives with respect to time and the material coordinate $\zeta$, respectively. For simplicity, we consider only the case, when the metric coefficients as well as the total pressure are not dependent on the $\zeta$ parameter.

In the particular case of a dipole magnetic field, we use the orthogonal dipole coordinates (see Fig. 1): the first one, $\lambda$, has constant values along magnetic field lines, the second one, $\sigma$, has constant values along the trajectories orthogonal to the magnetic field lines, and the third one, $\zeta$, is the azimuthal angle.

Finally, transformations of system Eq. (1)–(4) to the new variables and linearizations yield to equations for the parallel and perpendicular displacements as follows,

$$\frac{\partial (\sqrt{\tilde{g}_1} \tilde{\sigma})}{\partial t} + \frac{1}{\tilde{B}_z} \frac{\partial \tilde{P}}{\partial \tilde{\sigma}} = 0,$$

$$\frac{\partial (\tilde{g}_1 \tilde{\sigma})}{\partial t} - \frac{\partial (\tilde{g}_1 \tilde{\lambda})}{\partial \tilde{\sigma}} + \frac{1}{\tilde{g}_2 \tilde{\lambda}} \frac{\partial \tilde{g}_2}{\partial \tilde{\lambda}} \delta \tilde{P} = 0,$$

$$\frac{\partial (\tilde{g}_3 \tilde{\zeta})}{\partial t} - \frac{\partial (\tilde{g}_3 \tilde{\zeta})}{\partial \tilde{\lambda}} = 0,$$

where

$$\delta \tilde{P} = \frac{\gamma \tilde{\rho}_0}{\tilde{\rho}_0 (\gamma - 2)} [A_0 \delta \sigma + A_1 \delta \lambda - \sqrt{\tilde{g}_2 B_z} \delta \sigma_z],$$

$$A_0 = \left[ \frac{\partial \Pi}{\partial \sigma} - \frac{\partial \tilde{g}_2}{\partial \sigma} \frac{\tilde{B}_z}{2 \tilde{g}_2} \right],$$

$$A_1 = \left[ \frac{\partial \Pi}{\partial \lambda} - \frac{\partial \tilde{g}_2}{\partial \lambda} \frac{\tilde{B}_z}{2 \tilde{g}_2} \right].$$

Here $B_z$ is the normalized dipole magnetic field strength. Eq. (12) gives evidence that the plasma pressure perturba-
tions induced by the Alfvén wave are proportional to the background plasma beta.

3. Results

The numerical solution of Eqs. (9)–(11) is obtained on the base of finite difference Lax–Wendroff scheme. A dipole magnetic field is considered. The magnetic field lines are connected with the ionosphere characterized by a finite conductivity $\Sigma_i$. As shown in Fig. 1, the Alfvén wave pulse initiated at the equator ($A$) is propagating along the magnetic tube towards the ionosphere ($B$). At the ionospheric boundary we use the relationship between the magnetic field perturbation and the induced electric surface current $j_i$,

$$\frac{c}{4\pi} \mathbf{n} \times \delta \mathbf{B} = j_i, \quad (13)$$

where $\mathbf{n}$ is the normal unit vector. For simplicity, we assume the undisturbed magnetic field to be perpendicular to the ionospheric surface. Eq. (13) is based on the condition that the field-aligned currents induced by the Alfvén pulse are closed by the ionospheric surface current $j_i$ determined by the local Ohm’s law,

$$j_i = \Sigma_i \mathbf{E}. \quad (14)$$

Here $\mathbf{E}$ is the tangential electric field component which does not change across the ionospheric boundary. For the ideal conducting plasma in the magnetic tube, the electric field is proportional to the velocity perturbation,

$$\mathbf{E} = -\frac{1}{c} \mathbf{V} \times \mathbf{B}, \quad (15)$$

where $c$ is the speed of light, and $\mathbf{V}$ is the plasma velocity. For small perpendicular perturbations of the magnetic field and plasma velocity, Eqs. (13)–(15) yield

$$\mathbf{V} = \frac{c^2}{4\pi \Sigma_i \mathbf{B}} \delta \mathbf{B}. \quad (16)$$

The obtained boundary condition (16) is consistent with that derived by Leonovich and Mazur (1990) for more general case.

In dimensionless form, the boundary relationship (16) yields

$$\tilde{\mathbf{V}} = \mu \tilde{\mathbf{B}}, \quad (17)$$

where $\mu = c^2/(4\pi V_{ai} \Sigma_i)$, and $V_{ai}$ is the local Alfvén speed. This boundary condition determines reflections of incoming MHD waves.

At the equator, another boundary condition is given for the plasma velocity perturbation which is varying between positive and negative values during the initial short period, and after that, it is assumed to vanish.

Figs. 2, 3 show the propagation of the wave pulses along the dipole magnetic tube for two different wave polarizations.

Fig. 2 is corresponding to the meridional polarization of the velocity perturbation, and Fig. 3 is for the azimuthal polarization...
polarization. In both cases, the periods of the wave pulses are equal to $0.1 \frac{r_d}{V_{a0}}$. From top to bottom there are shown the magnetic field, the velocity, the energy flow through the tube, and the electric field perturbations, respectively, as functions of the radial distance for different time in units of $r_d/V_{a0}$, where $V_{a0}$ is the Alfvén speed corresponding to the equatorial point. The ratio $r_d/r_b$ is equal to 10.

Until the reflection zone, the velocity amplitude increases only slightly during the wave propagation. In the reflection zone, the amplitudes of the velocity and the magnetic field perturbations are decreasing functions of the radius $r$, and they are very small in the vicinity of the conducting boundary $r_b$. The electric field amplitude is obtained by multiplication of the plasma velocity amplitude and the dipole magnetic field strength, and it gives evidence of a monotonic increase until the conducting boundary.

Figs. 4 and 5 indicate the influence of the magnetic field convergence on the wave dissipation. Presented are the wave energy as a decreasing function of time in cases of meridional (Fig. 4) and azimuthal (Fig. 5) polarizations of the velocity perturbations for three different periods of wave pulses: $0.1 \frac{r_d}{V_{a0}}$ (top), $0.4 \frac{r_d}{V_{a0}}$ (medium), $1.0 \frac{r_d}{V_{a0}}$ (bottom). The time is scaled to the time interval of the Alfvén wave propagation from the equator to the ionosphere. The wave energy is constant during the propagation of the wave pulse until the boundary, and it decreases sharply after each reflection. The wave energy is normalized to its initial value. Dissipative properties of the conducting surface of the ionosphere are characterized by the parameter $\mu$ in formula (17) which is assumed to be 0.5. One can see from the figures that the decrease of the wave energy is much less pronounced for the case of meridional polarization, and for a larger time scale of the wave pulse.

4. Summary

The thin magnetic flux tube approach is applied for Alfvén wave propagation along converging magnetic field lines. The length scale of the Alfvén perturbations propagating along a narrowing magnetic tube increases proportionally to the magnetic field strength. The amplitudes of velocity and magnetic field perturbations do not change much while the wave does not arrive to a reflection region. This leads to a strong enhancement of the electric field amplitude in the course of the wave propagation in the direction of the magnetic pressure gradient. The reflection stage starts as soon as the length scale of the wave front becomes of the order of the distance from the boundary. The wave is reflecting rather from a very narrow part of the magnetic tube than from the conducting boundary in cases of sufficiently large wave length scales as well as large ratios of the maximal and minimal cross sections of the magnetic tube. In such cases, the wave energy flux to the conducting surface is rather small, and thus the dissipation of the wave perturbations is very weak. The length scale of a wave pulse is a crucial parameter for a wave propagation along a strongly narrowing magnetic flux tube. For shorter length scales, the dissipation of the wave energy is more pronounced. Our results can be applied for conditions at the Earth’s magnetosphere where Alfvén waves can be generated near the magnetopause by bursty reconnection of magnetic fields in cases of southward interplanetary magnetic field. These waves can propagate along dipole-like magnetic flux tubes towards the ionosphere which has a finite conductivity. Another application is that for Alfvén pulses produced by magnetic reconnection in the magnetotail and propagating towards Earth.

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